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Jen-Je Su

Te Kunenga
ki Pūrehuroa

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On the over-sizing problem of Dickey-Fuller-type tests with GARCH errors

Jen-Je Su♣

Department of Applied and International Economics
Massey University, Palmerston North
New Zealand

ABSTRACT

This paper examines the finite-sample accuracy (size) of a class of Dickey-Fuller-type tests in the presence of GARCH errors, with and without the influence from initial conditions of the underlying simulated path. Over-sizing is observed for all tests when the GARCH process is nearly degenerate and the volatility parameter is large, but the degree of size distortion varies across tests and is contingent on the initial condition. The standard Dickey-Fuller test is likely to over-reject more often than the modified tests, and while the over-sizing of the standard Dickey-Fuller test tends to be much more severe with the existence of the initial effect, the modified tests are fairly robust to that. The result due to the initial effect is linked to the size distortion causing by a sequence of small downward variance breaks arising in the early stage of the underlying process.

Keywords: GARCH; Unit root tests; Size distortion; Initial effect; Monte Carlo experiment

♣ Correspondence to: Department of Applied & International Economics, Private Bag 11 222, Massey University, Palmerston North, New Zealand. Email address: j.j.su@massey.ac.nz
Tel: 64 6 3505799 ext 2666
Fax: 64 6 350 5660

1. INTRODUCTION

It is a well known fact that the Dickey-Fuller (DF) test tends to over-reject the null of a unit root in the presence of GARCH errors. Kim and Schmidt (1993) first investigated the reliability of the DF test in this regard. They concluded that the DF test is generally over-sized and the problem becomes serious when the underlying GARCH process is nearly degenerate, nearly integrated and the volatility parameter is relatively large – that is, when the GARCH effect is "strong". Similar results were found in Haldrup (1994) and Lin et al. (2003). Valkanov (2005) also demonstrated that with strong GARCH effects while the small-sample DF distribution appears to converge to the asymptotic distribution, the convergence is rather slow. Because financial time series are typically characterized with strong GARCH effects, it might be suspicious about the result from standard practice of the DF test in empirical work.

The over-rejection problem appears in a class of modified DF tests too. According to Cook (2006), where the standard DF test and several modified DF tests were examined, while on the whole the modified tests do show some robust to GARCH, the advantage of using them over the standard test is not striking. In fact, when the sample size gets larger since the standard DF test statistic moves closer to its asymptotic distribution the advantage of the modified tests becomes even less noticeable. Also, like Kim and Schmidt (2003), Cook (2006) identified that the over-sizing of all DF-type tests is mainly driven by the volatility rather than the persistence of the variance process.

An important but commonly neglected or inadequately treated issue concerning the over-sizing problem for unit-root tests is the effect from initial values of a simulated I(1)-GARCH series. Kim and Schmidt (1993), Lin et al. (2003), Valkanov (2005) and Cook (2006) did not deal with such an effect in their Monte Carlo exercises. Haldrup (1994) is the only one in the unit root test literature that dealt with the effect. In order to get rid of the effect Haldrup (1994) suggested generating additional 10 observations for each simulating path and then deleted the first 10 from the path. Although such a rule seems working well in the elimination of the initial effect when the GARCH effect is moderate, it does not with strong GARCH. Actually, in a related literature regarding cointegration tests under GARCH, Lee and Tse (1996) argued that when the GARCH process is nearly integrated and nearly degenerate the simulated process takes a long while to settle down. In such a case discarding the first 500 observations is required to get rid of the initial effect. The same rule should apply to unit root tests as well.

In this paper, extensive Monte Carlo simulations are conducted to examine the influence from initial values of a simulated I(1)-GARCH process to several DF-type unit root tests. Basically, it is found that the over-rejection problem for the standard DF test with GARCH errors may become much more serious if the initial effect is not trimmed. In contrast, the initial effect does only minor impact to the modified tests. The result due to the mixed effect of GARCH and the initial conditions is linked to the literature of Kim et al. (2002) and Cook (2003, 2004) where the over-sizing problem for the DF-type tests is caused by an early break in innovation variance.

The paper proceeds as follows. In section 2, several DF-type tests are presented. Section 3 states design and reports results of the Monte Carlo experiment. Section 4 concludes.

2. DICKEY-FULLER-TYPE TESTS

Consider a model

$$y_t - \mu = \rho\left(y_{t-1} - \mu\right) + \varepsilon_t \,, \tag{1}$$

where y_t, t = 1,2, …, T, are observations, ε_t is a serially-uncorrelated zero-mean process, and μ, ρ are unknown parameters. Let $\dot{y}_t = y_t - \bar{y}$ and $\bar{y} = T^{-1}\sum_{t=1}^{T} y_t$. The standard DF test tests the unit root hypothesis $H_0 : \rho = 1$ against the alternative $H_1 : |\rho| < 1$ using a test statistic

$$\tau = \left(\hat{\rho} - 1\right) / se(\hat{\rho}) \tag{2}$$

where $\hat{\rho} = \sum_{t=2}^{T} \dot{y}_t \dot{y}_{t-1} / \sum_{t=2}^{T} \dot{y}_{t-1}^2$ is the OLS estimator of ρ in (1) and $se(\hat{\rho})$ is its standard error. Despite its popularity, the DF test is known to have low power when ρ is close to one.

Several attempts have been made to improve the power of the standard DF test in the literature. Pantula et al. (1994) introduced a weighted symmetric (WS) estimator

$$\hat{\rho}_{ws} = \sum_{t=2}^{T} \dot{y}_t \dot{y}_{t-1} / (\sum_{t=2}^{T-1} \dot{y}_t^2 + T^{-1}\sum_{t=1}^{T} \dot{y}_t^2) \tag{3}$$

of ρ in (1). Based on the WS estimator, they suggested a modified DF test (DF-WS):

$$\tau_{WS} = \sigma_{ws}^{-1}\left(\hat{\rho}_{ws} - 1\right)\left(\sum_{t=2}^{T-1} \dot{y}_t^2 + T^{-1}\sum_{t=1}^{T} \dot{y}_t^2\right)^{1/2} \tag{4}$$

where $\sigma_{ws}^2 = (T-2)^{-1}[\sum_{t=1}^{T-1} w_t (\dot{y}_t - \hat{\rho}_{ws}\dot{y}_{t-1})^2 + \sum_{t=2}^{T} (1 - w_{t+1})(\dot{y}_t - \hat{\rho}_{ws}\dot{y}_{t+1})^2]$ with $w_t = (t-1)/T$. Basically, the power gain of this test comes from the utilization of the reverse autoregressive (AR) representation in addition to the forward AR representation.

Power improvement may also be achieved using different mean-adjustment schemes other than OLS as in the standard DF test. Elliot et al (1996) proposed a modified test using the application of local-to-unity demeaning via generalized least squares (GLS) estimation. The locally demeaned series $\ddot{y}_t$ is derived as $\ddot{y}_t = y_t - \hat{\beta}$ where $\hat{\beta}$ is obtained by regressing $y_\alpha = (y_1, y_2 - \alpha y_1, ..., y_T - \alpha y_{T-1})'$ on $z_\alpha = (1, 1-\alpha, ..., 1-\alpha)'$ with $\alpha = 1 - (7/T)$. The resulting unit root test, denoted as DF-GLS, is the t-ratio of ϕ from the regression: $\Delta\ddot{y} = \phi\,\ddot{y}_{t-1} + \varepsilon_t$. On the other hand, Shin and So (2001) suggested the use of recursively adjusted mean $\bar{y}_t = t^{-1}\sum_{i=1}^{t} y_i$. The recursively mean-adjusted version of DF test (DF-REC) has the same format as (2), except that $\hat{\rho}$ and $se(\hat{\rho})$ are obtained from the regression of $(y_t - \bar{y}_{t-1})$ on $(y_{t-1} - \bar{y}_{t-1})$.

While the power advantage of the above mentioned modified DF tests has been confirmed in subsequently studies and become recognized for applied researcher (see, for example, Leybourne et al. (2005)), their robustness in the presence GARCH has been discussed only very recently in Cook (2006).

3. MONTE CARLO EXPERIMENT AND RESULTS

The data-generating process to be considered is a driftless integrated process with GARCH innovations. The process is

$$y_t = y_{t-1} + \varepsilon_t, \quad t=1,\ldots,T+d, \tag{5}$$

where ε_t is assumed to be a GARCH process: $\varepsilon_t = h_t \eta_t$, where η_t is i.i.d. N(0,1),

$$h_t = \phi_0 + \phi_1 \varepsilon_{t-1}^2 + \phi_2 h_{t-1}. \tag{6}$$

GARCH parameters – typically, ϕ_1 is know as the volatility parameter and ϕ_2 the persistence parameter – are set to reflect the empirical findings. Specifically, ϕ_1 is set between 0 and 0.349 and ϕ_2 between 0.65 and 0.95, but $\phi_1 + \phi_2$ never goes larger than 0.999, and ϕ_0 is calibrated to make the unconditional variance equal 1 in all cases: $\phi_0 = 1 - \phi_1 - \phi_2$. The initial value of the unit-root process is set $y_0 = 0$ and the initial variance $h_0 = 1$. The pseudo random innovations $\{\varepsilon_t\}_{t=1}^{T+d}$ are drawn from $N(0, h_t)$ and used to construct the unit-root process for T=100, 500, 1000, 2000 and d=500. Two sets of series are used for unit-root testing: series [1] contains observations from 1 to T, and series [2] from d+1 to T+d. In the first series, the initial effect is left untreated while the second series, according to Lee and Tse (1996), the initial effect is off-loaded.[1] All simulations are based on 10,000 replications and done by GAUSS. All rejection frequencies are calculated at the nominal 5% significance level.[2] The Monte Carlo results are reported in Table 1.

The results in Table 1 are summarized and commented as follows. First, the simulation result shows that in some cases the difference of size distortion between the standard DF test and the modified tests can be substantially large. In particular, as the sum of GARCH parameters approaches extremely close to the boundary of integration and the volatility parameter is relatively large (say, $\phi_1 + \phi_2 \geq 0.99$ and $\phi_1 \geq 0.24$), while the over-sizing problem of the standard DF test deteriorates considerably the modified tests only get worse slightly. For example, when $(\phi_1, \phi_2) = (0.349, 0.65)$ and T=500, the rejection frequency of the

[1] The appropriateness of this elimination rule has also been confirmed by the author using simulation with a wide class of GARCH models. The result is available upon request

[2] Critical values are obtained by simulation with 50,000 draws for each test at different sample sizes and available upon request.

standard DF test is 0.358, but 0.087, 0.146 and 0.109 for DF-GLS, DF-WS and DF-REC, respectively. Also, the tests tend to show different patterns of size distortion corresponding to T. The standard DF test tends less distorted when T is larger if $\phi_1 + \phi_2 \le 0.995$ and becomes irregular if $\phi_1 + \phi_2 = 0.999$; the DF-GLS test seems unaffected with T; the DF-WS and the DF-REC tests appear more and then less distorted as T gets larger.

Second, the Monte Carlo result shows that the influence from initial conditions for the data generation process may contribute a significant proportion of size distortion for the standard DF test. Indeed, in some cases, the over-rejecting problem due to GARCH might look much more serious than it actually is provided that such an influence has been eliminated. For example, the rejection rate drops from 0.358 to 0.108 for $(\phi_1, \phi_2) = (0.349, 0.65)$ at T=500 when the effect is gone. Accordingly, failing to notice the initial effect the severe over-rejection of the standard DF test with strong GARCH errors is likely to be overstated. On the contrary, the initial effect does not inflate size of the DF-GLS and the DF-REC tests and only causes a bit more size distortion for the DF-REC test. Besides, without the initial effect all tests seem very stable over a wide range of sample size, implying that the tests converge to the asymptotic distribution rather fast.

Third, even if the initial effect is removed, the GARCH effect itself still brings about some size distortion for the DF-type tests. Again, since the unconditional kurtosis increases as ϕ_1 increases when $\phi_1 + \phi_2$ is fixed, given the value of $\phi_1 + \phi_2$, the size distortion is normally bigger with a larger ϕ_1.[3] Size improvement of the modified DF tests (with the DF-REC test an exception) over the standard DF test is obvious, but not very striking. Among the tests, the DF-GLS test appears to be the most robust one. In the presence of GARCH, the DF-GLS test tends to over-reject but the rejection frequency never goes larger than 0.088 and this happens only when the GARCH effect is extremely strong.

Finally, it is worth noting that the simulation result due to the initial effect can actually be related to the over-sizing problem for the same class of unit-root tests considered previously when there exists a variance break. According to Kim et al. (2002), the standard DF test tends severely over-sized when applied to a unit-root process experiencing an abrupt decrease in innovation variance if the decrease is large and occurs in the early stage of the process. In contrast, as shown in Cook (2003, 2004), the modified DF tests seem quite robust to such a break. As for the case of GARCH, Lee and Tse (1996) noted that when a GARCH model is nearly degenerate and with a relative large volatility parameter the initial variance $h_0 = 1$ is too far in the right tail of its stationary distribution, so the simulated time-varying variance h_t tends to decline as t gets larger and this declining will last for a while. In other words, if a simulated GARCH series is untrimmed it will behave like a path arisen from a setting where innovation variance undergoes a sequence of downward breaks soon after the start and the breaks become larger and last longer when the GARCH process is closer to degenerate and with a larger

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[3] The unconditional kurtosis for an GARCH(1,1) is $3[1-(\phi_1+\phi_2)^2 - 2\phi_1^2]^{-1}[1-(\phi_1+\phi_2)^2]$.

volatility parameter. This should help to explain the puzzling size properties regarding the standard DF test and the modified tests in the presence of GARCH, with and without the initial effect. Of course, Kim et al. (2002) and Cook (2003, 2004) obtain their results only concerning a sudden large drop in variance, but it is not unreasonable to expect that similar results should occur for a series of small but prolonging variance reduction arising in the early stage of the unit-root path. As a matter of fact, according to the simulation result (see Appendix I), this is indeed the case.

4. CONCLUSION

In this paper, extensive Monte Carlo simulations are carried out to study the size performance of a class of Dickey-Fuller tests in the presence of GARCH errors, with and without the influence from initial values of the underlying process. In addition to the standard DF test, three modified DF tests are considered. Basically, simulation results show that the compound effect from GARCH and initial conditions can cause significant upward size distortion for the standard DF test but the problem is much less severe if the effect is solely from GARCH. In contrast, the modified tests seem insensitive to the initial effect. Even if the initial effect has been suitably controlled, all tests suffer size distortion causing by GARCH to some degree. Among the tests, the DF-GLS test seems to have least size distortion.

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Appendix I

To study the empirical size of the DF-type tests with breaks in innovation variance, a unit-root process is generated according to equation (5) with $\varepsilon_t = \eta_t \sigma_t$, where η_t is an i.i.d. N(0,1) and σ_t is the scale of variance at t: for an abrupt break

$$\sigma_t = \begin{cases} \sigma_1 & for \ \ t \le T_1, \\ \sigma_2 & for \ \ T_1 < t, \end{cases}$$

and for a series of smooth breaks between T_1 and T_2,

$$\sigma_t = \begin{cases} \sigma_1 & for \ \ t \le T_1, \\ \sigma_1 + \left(\dfrac{\sigma_2 - \sigma_1}{T_2 - T_1} \right) \left(t - T_1 \right) & for \ \ T_1 < t \le T_2, \\ \sigma_2 & for \ \ T_2 < t. \end{cases}$$

Denoting the break ratio (σ_2 / σ_1) as δ, the values $\delta \in \{0.25, 0.4, 0.6\}$ are considered. Four cases of breaks are examined: CASE I assumes a single big break arising right after T_1; CASE II~CASE IV witness a sequence of small breaks of same size ($((\sigma_2 - \sigma_1)/(T_2 - T_1))$) between T_1 and T_2. Breaks start at either T_1=1 or T_1=0.1*T+1, and for CASE II~CASE IV they end at T_2= T_1+0.1*T-1 (CASE II), T_2= T_1+0.2*T-1 (CASE III), or T_2= T_1+0.3*T-1 (CASE IV), where T is either 100 or 500. Since the initial effect is not a concern, d=0 in (5). The simulation result of rejection frequency for the DF-type tests at the 5% significance level is given in Table 2.

From Table 2, it is clear that early downward breaks cause significant size distortion for the standard DF test regardless the breaks are one-shot or sequential. Actually, sequential small breaks can cause more size distortion than an abrupt large break if the breaks begin at T_1=1 while the opposite is true if the breaks occur later. On the other hand, variance breaks of all sorts bring about much less serious size problems for the modified DF tests.

Table 1: Rejection frequency at 5% level in the presence of GARCH

[a] T=100

(ϕ_1,ϕ_2)	$\phi_1+\phi_2$	Initial effect untreated				Initial effect off-loaded			
		DF	DFGLS	DFWS	DFREC	DF	DFGLS	DFWS	DFREC
(0.300,0.650)	0.950	0.092	0.067	0.065	0.069	0.080	0.070	0.067	0.075
(0.200,0.750)	0.950	0.077	0.063	0.061	0.066	0.071	0.066	0.064	0.067
(0.100,0.850)	0.950	0.059	0.058	0.052	0.055	0.062	0.055	0.053	0.055
(0.000,0.950)	0.950	0.053	0.049	0.048	0.051	0.048	0.050	0.047	0.050
(0.340,0.650)	0.990	0.173	0.079	0.088	0.081	0.095	0.080	0.078	0.086
(0.240,0.750)	0.990	0.127	0.067	0.070	0.065	0.087	0.074	0.071	0.076
(0.140,0.850)	0.990	0.087	0.057	0.058	0.057	0.072	0.064	0.062	0.067
(0.040,0.950)	0.990	0.055	0.053	0.049	0.054	0.055	0.050	0.049	0.053
(0.345,0.650)	0.995	0.223	0.077	0.091	0.071	0.099	0.080	0.083	0.090
(0.245,0.750)	0.995	0.168	0.068	0.075	0.060	0.093	0.072	0.068	0.075
(0.145,0.850)	0.995	0.101	0.059	0.059	0.056	0.076	0.063	0.063	0.067
(0.045,0.950)	0.995	0.056	0.049	0.050	0.052	0.053	0.051	0.045	0.049
(0.349,0.650)	0.999	0.361	0.097	0.128	0.084	0.100	0.079	0.079	0.084
(0.249,0.750)	0.999	0.250	0.082	0.097	0.070	0.094	0.074	0.074	0.079
(0.149,0.850)	0.999	0.135	0.064	0.065	0.053	0.080	0.058	0.055	0.060
(0.049,0.950)	0.999	0.063	0.058	0.054	0.056	0.056	0.055	0.052	0.053

[b] T=500

(ϕ_1,ϕ_2)	$\phi_1+\phi_2$	Initial effect untreated				Initial effect off-loaded			
		DF	DFGLS	DFWS	DFREC	DF	DFGLS	DFWS	DFREC
(0.300,0.650)	0.950	0.077	0.065	0.072	0.075	0.075	0.063	0.070	0.074
(0.200,0.750)	0.950	0.071	0.059	0.068	0.068	0.069	0.059	0.066	0.067
(0.100,0.850)	0.950	0.055	0.054	0.053	0.054	0.056	0.052	0.051	0.052
(0.000,0.950)	0.950	0.048	0.045	0.049	0.047	0.050	0.050	0.050	0.049
(0.340,0.650)	0.990	0.142	0.073	0.093	0.088	0.101	0.080	0.094	0.101
(0.240,0.750)	0.990	0.123	0.065	0.081	0.081	0.092	0.079	0.086	0.088
(0.140,0.850)	0.990	0.090	0.065	0.075	0.074	0.084	0.069	0.075	0.077
(0.040,0.950)	0.990	0.054	0.053	0.053	0.052	0.055	0.056	0.056	0.056
(0.345,0.650)	0.995	0.188	0.077	0.102	0.098	0.104	0.082	0.093	0.098
(0.245,0.750)	0.995	0.174	0.077	0.104	0.093	0.104	0.078	0.091	0.097
(0.145,0.850)	0.995	0.133	0.069	0.080	0.073	0.090	0.069	0.076	0.078
(0.045,0.950)	0.995	0.060	0.053	0.054	0.051	0.065	0.058	0.056	0.057
(0.349,0.650)	0.999	0.358	0.087	0.146	0.109	0.108	0.082	0.099	0.102
(0.249,0.750)	0.999	0.336	0.082	0.130	0.094	0.105	0.084	0.098	0.102
(0.149,0.850)	0.999	0.239	0.075	0.101	0.075	0.096	0.068	0.080	0.081
(0.049,0.950)	0.999	0.089	0.055	0.061	0.057	0.069	0.053	0.057	0.057

[c] T=1000

(ϕ_1,ϕ_2)	$\phi_1+\phi_2$	Initial effect untreated				Initial effect off-loaded			
		DF	DFGLS	DFWS	DFREC	DF	DFGLS	DFWS	DFREC
(0.300,0.650)	0.950	0.076	0.062	0.074	0.083	0.077	0.070	0.074	0.082
(0.200,0.750)	0.950	0.064	0.063	0.067	0.073	0.068	0.062	0.065	0.073
(0.100,0.850)	0.950	0.057	0.054	0.055	0.060	0.058	0.054	0.057	0.064
(0.000,0.950)	0.950	0.049	0.055	0.050	0.053	0.054	0.054	0.051	0.055
(0.340,0.650)	0.990	0.127	0.077	0.094	0.100	0.104	0.084	0.099	0.109
(0.240,0.750)	0.990	0.115	0.074	0.089	0.094	0.096	0.083	0.089	0.101
(0.140,0.850)	0.990	0.081	0.065	0.073	0.079	0.080	0.067	0.076	0.081
(0.040,0.950)	0.990	0.058	0.054	0.056	0.061	0.062	0.053	0.057	0.060
(0.345,0.650)	0.995	0.160	0.083	0.108	0.109	0.103	0.088	0.098	0.110
(0.245,0.750)	0.995	0.153	0.080	0.102	0.105	0.107	0.084	0.096	0.107
(0.145,0.850)	0.995	0.120	0.070	0.087	0.092	0.094	0.074	0.085	0.093
(0.045,0.950)	0.995	0.066	0.057	0.059	0.066	0.062	0.059	0.058	0.064
(0.349,0.650)	0.999	0.302	0.083	0.130	0.115	0.115	0.084	0.101	0.116
(0.249,0.750)	0.999	0.304	0.092	0.131	0.113	0.108	0.083	0.097	0.109
(0.149,0.850)	0.999	0.243	0.079	0.109	0.097	0.106	0.081	0.090	0.099
(0.049,0.950)	0.999	0.106	0.060	0.068	0.068	0.081	0.063	0.064	0.068

[d] T=2000

(ϕ_1,ϕ_2)	$\phi_1+\phi_2$	Initial effect untreated				Initial effect off-loaded			
		DF	DFGLS	DFWS	DFREC	DF	DFGLS	DFWS	DFREC
(0.300,0.650)	0.950	0.065	0.058	0.064	0.068	0.069	0.057	0.065	0.070
(0.200,0.750)	0.950	0.060	0.056	0.057	0.057	0.058	0.058	0.058	0.060
(0.100,0.850)	0.950	0.055	0.056	0.057	0.057	0.056	0.050	0.052	0.053
(0.000,0.950)	0.950	0.046	0.051	0.049	0.049	0.050	0.050	0.051	0.050
(0.340,0.650)	0.990	0.108	0.078	0.091	0.095	0.094	0.074	0.092	0.098
(0.240,0.750)	0.990	0.102	0.073	0.089	0.091	0.091	0.076	0.087	0.093
(0.140,0.850)	0.990	0.083	0.064	0.073	0.077	0.081	0.068	0.074	0.080
(0.040,0.950)	0.990	0.054	0.050	0.052	0.053	0.054	0.053	0.054	0.053
(0.345,0.650)	0.995	0.128	0.076	0.094	0.097	0.105	0.083	0.094	0.103
(0.245,0.750)	0.995	0.128	0.080	0.098	0.101	0.103	0.080	0.093	0.099
(0.145,0.850)	0.995	0.106	0.075	0.087	0.089	0.083	0.070	0.080	0.085
(0.045,0.950)	0.995	0.065	0.057	0.057	0.059	0.059	0.058	0.059	0.059
(0.349,0.650)	0.999	0.229	0.080	0.112	0.101	0.106	0.082	0.099	0.110
(0.249,0.750)	0.999	0.231	0.079	0.111	0.101	0.107	0.088	0.103	0.112
(0.149,0.850)	0.999	0.214	0.082	0.103	0.095	0.109	0.087	0.098	0.104
(0.049,0.950)	0.999	0.104	0.061	0.071	0.067	0.086	0.065	0.069	0.069

Table 2: Rejection frequency at the 5% level with variance breaks

[a] Variance breaks starting at T_1=1

		T=100				T=500			
		DF	DFGLS	DFWS	DFREC	DF	DFGLS	DFWS	DFREC
δ=0.25	CASE I	0.149	0.041	0.046	0.039	0.073	0.043	0.048	0.046
	CASE II	0.366	0.076	0.093	0.048	0.367	0.064	0.095	0.051
	CASE III	0.379	0.086	0.107	0.064	0.373	0.080	0.113	0.064
	CASE IV	0.323	0.089	0.108	0.066	0.315	0.083	0.113	0.064
δ=0.4	CASE I	0.088	0.046	0.048	0.048	0.057	0.047	0.049	0.046
	CASE II	0.184	0.051	0.057	0.041	0.180	0.048	0.059	0.040
	CASE III	0.215	0.055	0.063	0.042	0.209	0.055	0.065	0.045
	CASE IV	0.208	0.061	0.066	0.047	0.203	0.059	0.068	0.046
δ=0.6	CASE I	0.069	0.050	0.048	0.051	0.049	0.044	0.045	0.045
	CASE II	0.087	0.046	0.047	0.047	0.074	0.040	0.045	0.042
	CASE III	0.107	0.045	0.049	0.047	0.094	0.041	0.046	0.041
	CASE IV	0.117	0.046	0.050	0.046	0.102	0.040	0.048	0.040

[b] Variance breaks starting at T_1=0.1*T+1

		T=100				T=500			
		DF	DFGLS	DFWS	DFREC	DF	DFGLS	DFWS	DFREC
δ=0.25	CASE I	0.400	0.079	0.102	0.055	0.406	0.080	0.120	0.061
	CASE II	0.398	0.095	0.116	0.066	0.399	0.084	0.124	0.073
	CASE III	0.336	0.095	0.114	0.070	0.331	0.083	0.117	0.071
	CASE IV	0.267	0.083	0.099	0.069	0.260	0.077	0.108	0.068
δ=0.4	CASE I	0.228	0.057	0.066	0.045	0.225	0.053	0.067	0.045
	CASE II	0.239	0.066	0.072	0.048	0.230	0.057	0.072	0.046
	CASE III	0.223	0.066	0.073	0.051	0.213	0.060	0.073	0.045
	CASE IV	0.199	0.069	0.075	0.053	0.190	0.058	0.072	0.047
δ=0.6	CASE I	0.122	0.050	0.049	0.043	0.116	0.051	0.055	0.044
	CASE II	0.126	0.050	0.050	0.043	0.120	0.050	0.058	0.044
	CASE III	0.128	0.050	0.052	0.043	0.122	0.053	0.058	0.045
	CASE IV	0.131	0.051	0.054	0.044	0.120	0.053	0.059	0.044

Note. CASE I: a one-shot variance break at T_1; CASE II: sequential breaks starting at T_1 and ending at $T_2=T_1$+0.1*T-1; CASE III: sequential breaks starting at T_1 and ending at $T_2=T_1$+0.2*T-1; CASE IV: sequential breaks starting at T_1 and ending at $T_2=T_1$+0.3*T-1.

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