The Macroeconomics of Stock Prices in the Medium Term and in the Long Run

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Abstract. This paper develops a macro-model of stock prices that predicts that the growth rates in real stock prices and real dividends gravitate toward predictable constants in the long run, but fluctuate on approximately decennial frequencies due to movements in capital’s income share. The model can be used to analyse the effects on stock prices of technology shocks, supply shocks, imperfections in the credit markets, change in taxes, and whether stocks are less risky in the long run than in the short run. Using macroeconomic data over 130 years for 22 OECD countries, the data give support for the model.

JEL Classification: E44, G12.

Key words: Stock prices and dividends in the long run, stock valuation, required stock returns, and macroeconomic factors.

1 Introduction

Based on the Tobin’s q model of investment this paper develops a macroeconomic framework that is used to explain the medium-term and long-run movements in real stock prices and dividends. It is shown that movements in labour’s income share and the capital output ratio dominate medium-term movements in real stock prices and dividends. In the absence of retained earnings, real stock prices and real dividends converge to constant means in the long run because of the Tobin’s q effect. If a fraction of earnings is retained within the company the real stock prices and dividends will, in the long run, grow at predictable constants independent of key macroeconomic aggregates such as growth in GDP and labour productivity.

The model integrates the labour market into the investment models of Eisner and Strotz (1963), Lucas (1967), Gould (1968), Foley and Sidrauski (1970) and gives explicit attention to the interaction between the stock market, fixed investment and the labour market. Extending the models of Summers (1981) and Madsen and Davis (2006) to allow for the influence of the labour market on

1 Helpful comments and suggestions from Christian Groth, Søren Johansen, Axel Mossin, seminar participants at the University of Copenhagen, Brunel University, University of Odense, University of Western Australia, Copenhagen Business School, Aarhus Business School, and University of Konstantz, and particularly, two referees are gratefully acknowledged.
stock prices focusing specifically on the movement in real stock prices in the long run. The novelty of the paper is to show that equilibrium in the markets for fixed investment and labour provide a rationale for mean reversion in real share prices and earnings per unit of capital.

The model seeks to fill the gap that prevails in the empirical literature on mean reversion in stock prices, and to some extent, also stock returns and earnings per unit of capital. Poterba and Summers (1988) and Fama and French (1988) have shown that real stock prices revert toward constant means Fama and French (2000) and, to some extent, also Chan et al. (2000) demonstrate that earnings per unit of capital are mean reverting. Furthermore, Shiller (2001, p 253), and implicitly Smithers and Wright (2000), argue that earnings per unit of capital are mean reverting and that stock prices converge to a mean that is trending upwards over time. In this literature the forces that are responsible for the mean reversion remain, to a large extent, unexplained.2

Using data for 22 OECD countries over 130 years, the movements and the interaction between the output-capital ratio, earnings per unit of capital, real dividends and real stock prices are examined using graphical illustrations and econometric analysis. It is shown that labour’s share in total income is a key determinant of the movements in real stock prices and real dividends on decennial frequencies. Furthermore, ex post real stock returns are found to gravitate towards a constant level of about 7% in the long run, and the growth in real stock prices and real dividends converge to a constant of about 3%.

2 Long run evidence on movements in corporate earnings

The basic idea in this paper is that earnings per unit of capital fluctuate on medium term frequencies due to fluctuations in income shares and capital stock, but converge to constant means in the long run under some regularity assumptions that are detailed below. Figure 1 displays the unweighted average profit rate for five industrialized countries over the past 133 years (USA, UK, Japan, Italy, and Canada). The profit rate is estimated as earnings divided by non-residential capital stock at current prices. All five countries are included in the figure after 1929, but only the UK and Italy are included in the data before 1929. Since the statistics on capital stock have a broader coverage than profits, the figures on returns to capital are on the lower side. The data sources are listed in the data appendix.

Earnings are estimated as revenue minus labour costs, depreciation and interest payments for all countries except for Italy. Gross operating surplus is estimated as value added income minus labour income and, as such, contains interest payments on debt and depreciation. Operating surplus exaggerate corporate earnings as interest payments on debt are not a part of corporate earnings. Thus, the profit rate is for Italy adjusted to have the same mean as the average of the other countries. The problem associated with this procedure is that the share of interests on debt in total operating surplus

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2 The model of Campbell and Cochrane (1999) implies mean reversion of share returns on business cycle frequencies, however, not on medium term and long term frequencies, which is the focus of this paper.
need not be constant because of the following: First, the debt-capital ratio may change over time, particularly in periods of rapid changing government debt. Second, the ratio of nominal returns on debt and capital may change over time. Since the movements in the computed profit rates for the UK and Italy are quite similar for the UK and Italy over the entire period from 1870 to 2002 the share of interests on debt in total operating surplus is likely to not have been too unstable.

The figure shows that the profit rate fluctuates about a mean of about 6.5% on medium term frequencies, however, it appears that the mean has been reduced slightly in the post-WWII period; an issue that is addressed later in the paper. Returns were reduced substantially during the interwar period because strong labour movements increased labour’s share of total income and particularly because of the Great Depression. The surge in profits from 1940 to 1943 was a result of increasing demand coupled by a sharp reduction in investment activity in the 1930s, which reduced the capital stock and hence increased returns per unit of capital. The reduction in the profit rate throughout the 1970s was caused by a strong labour movement that, in conjunction with adverse supply shocks, increased labours’ income share and hence squeezed profits (Madsen, 1998). Labour’s income share has since been reduced substantially, which has paved the way for the profit recovery.

The path in the profit rate over the past 133 years has the following three implications for the medium term and the long term movements in real stock returns. First, the tendency for the profit rate to converge to an almost constant mean in the long run implies that real stock prices and dividends do not grow over time because of GDP or productivity growth induced profit, as commonly assumed. However, as earnings are retained within the company the constant earnings per unit of capital gradually become concentrated on fewer shares over time. Second, medium-term
movements in the profit rate are predominantly a result of changes in labour’s income share and the capital stock. Third, the profit rate is pro-cyclical due to asymmetrical adjustment costs in investment and, to some extent, also employment. These issues are addressed in more detail in the next sections.

3 A macroeconomic model of stock prices

The model seeks to explain the observations from the previous section that the profit rate 1) fluctuates on medium term frequencies; but 2) converges to a constant or slow moving mean in the long run. Pro-cyclical profit rates are also implied by the model; however, the demand side is not formally considered. The model consists of the stock market, the market for fixed investment; and the labour market. To simplify the analysis, and without the loss of generality, the labour market is omitted from the analysis in the first part of this section. This can be done because the model is block recursive; there is no feedback effect from the markets for stock and fixed investment to the labour market. The model is augmented with the labour market in Sections 3.3 and 3.4.

Investment and stock prices are determined jointly from the following optimization problem of the firm, which under the Cobb-Douglass technology assumption, is given by:

$$\max \Pi = \int_{t=0}^{\infty} e^{-\rho t} \left\{ (1 - \tau(t)) \left[ A(t)K^\alpha(t)L^{1-\alpha}(t) - W(t)L(t) - C(I(t)) \right] - I(t)(1 - \Gamma(t)) \right\} dt$$

st. $\dot{K}(t) = I(t) - \delta K(t)$

where $\Pi$ is the present value of real free cash flow, $K$ is capital services, $I$ is gross investment, $W$ is the real wage rate, $L$ is labour services, $\rho$ is a constant required return to equity, $C(I)$ is the adjustment cost of investment, $C(0) = 0$, $C'(0) = 0$, $C''(\bullet) > 0$, $\delta$ is the rate of capital depreciation, $\tau$ is the corporate tax rate, $A$ and $\alpha$ are fixed parameters, and $\Gamma$ is the sum of investment tax credits as a percentage of acquisition costs and the present value of expected future depreciation allowances for tax purposes as a percentage of acquisition costs. A dot over a variable signifies first differences. The firm is an all equity firm and all the free cash flow is paid out. Note that $\Pi$ is often referred to as discounted real profits (see for example Romer, 2006, Chapter 8), however, $\Pi$ is defined here as the present value of real free cash flow as in the corporate finance literature (see for example Brealey and Meyers, 1996).3

The current-value Hamiltonian of this optimisation problem is given by:

$$\mathcal{H}(t) = (1 - \tau(t))[A(t)K^\alpha(t)L^{1-\alpha}(t) - W(t)L(t) - C(I(t))] - I(t)(1 - \Gamma(t))$$

3 The following conventions are used in this paper: free cash flow = $[A(t)K^\alpha(t)L^{1-\alpha}(t) - W(t)L(t) - C(I(t)))] - I(t)(1 - \Gamma(t))$, earnings = $[A(t)K^\alpha(t)L^{1-\alpha}(t) - W(t)L(t)]$, and revenue = $A(t)K^\alpha(t)L^{1-\alpha}(t)$. 
where \( Q \) is the shadow price of capital or Tobin's \( q \). The first order conditions for optimum under the assumption of imperfect competition are:

\[
(1 - \tau(t)) \alpha^Y(t) \left( 1 + \frac{1}{\eta(t)} \right) = (\rho + \delta) Q(t) - \dot{Q}(t) \tag{2}
\]

\[
\lim_{t \to \infty} e^{-\rho t} Q(t) K(t) = 0 \tag{3}
\]

\[
1 - \Gamma(t) + C'(I(t))(1 - \tau(t)) = Q(t) \tag{4}
\]

\[
(1 - \alpha) \frac{Y(t)}{K(t)} \left( 1 + \frac{1}{\eta(t)} \right) = W(t)
\]

where \( \eta \) is the price elasticity of demand facing the firm and \( Y \) is output.

To find the relationship between Tobin’s \( q \) and stock prices integrate Equation (2) and use the transversality condition given by Equation (3) to get:

\[
Q(t) = \int_{s=t}^{\infty} e^{-\rho(s-t)} (1 - \tau(t)) \alpha^Y(t) \left( 1 + \frac{1}{\eta(t)} \right) ds. \tag{5}
\]

This equation shows that Tobin’s \( q \) equals discounted after-tax marginal profits. Since stock prices in the standard dividend discount framework are determined by discounted dividends it follows that Tobin’s \( q \) equals the stock prices where stock prices are normalized by capital stock.

Below the marginal profits are measured as after-tax earnings per unit of capital, which is total revenue minus labour costs per unit of capital at the margin. This approximation requires the production function is linearly homogenous and that technological progress is disembodied so that the marginal profit does not depend on the dates at which the capital is installed. The latter assumption is relaxed below. Following Lucas (1967) and Abel and Blanchard (1986) consider the firm’s profit maximization problem under the assumption of perfect competition: \( \pi(K,I) = \max[AK^\alpha L^{1-\alpha} - WL - C(I)] \). From Eulers theorem it follows that \( \pi(K,I) = \alpha^Y K - C(I) \) and that \( \frac{\partial \pi}{\partial K} = \alpha^Y K \). This establishes that the marginal profit, or marginal product, of capital is equal to the average product of capital, \((Y-WL)/K\), or earnings per unit of capital.

### 3.1 Equilibrium under perfect competition

Under the assumptions of perfect competition in the goods and labour markets, returns to capital are independent of the labour market. Stock returns are then determined by the interaction between the
market for fixed investment and the stock market. For the representative firm Equations (2)-(4) form the following simultaneous first-order differential equation system:

\[
\dot{Q}(t) = (\rho + \delta)Q(t) - \left(1 - \tau(t)\right)\alpha \frac{Y(t)}{K(t)},
\]

\[
\dot{K}(t) = F \left[\frac{Q(t) + \Gamma(t) - 1}{(1 - \tau(t))}\right],
\]

where \( \eta \to \infty \). Equation (6) is an ordinary equity market equilibrium condition indicating that the required returns to equity are equal to expected capital gains plus the marginal productivity of capital. Equation (7) is Tobin’s \( q \) model of investment where \( I = 0 \) for \( [Q(t) + \Gamma(t) - 1] = 0 \), \( I > 0 \) for \( [Q(t) + \Gamma(t) - 1] > 0 \), and \( I < 0 \) for \( [Q(t) + \Gamma(t) - 1] < 0 \). Tax credits and depreciations for tax purposes, reduce the effective acquisition cost of capital and, therefore, the benchmark level of \( q \) at which investment is undertaken. Figure 2 displays the dynamics of the two equations, where the \( \dot{Q} = 0 \) schedule slopes downwards due to the assumption of diminishing returns to capital.

An important property of the model is that the stock price converges to a constant mean in the long run resulting from an innovation in the marginal productivity of capital or a shift in the required stock returns. The model implies that the marginal productivity of capital is constant in steady state. Solving (6) and (7) in steady state yields:

\[
\alpha \frac{Y(t)}{K(t)} = \frac{1 - \Gamma(t)}{1 - \tau(t)} (\rho + \delta),
\]

where a bar over a variable indicates that it is a steady state value. Since required returns and taxes are constant in steady state it follows that the marginal productivity of capital is constant in steady state. This implies that after-tax earnings per unit of capital are constant in the long run. More intuitively consider the dynamic adjustment towards steady state following a technology shock. In a no tax world this equation shows the well-known fact that the marginal productivity of capital is equal to stock returns plus the rate of depreciation.

Figure 2 shows the movement in stock prices towards their steady state following a shock. Suppose that the system is initially in its long run equilibrium and an unanticipated positive technology shock increases the marginal productivity of capital and, consequently, shifts the \( \dot{Q} = 0 \) schedule to the right in Figure 2. The perfect foresight stock market instantaneously jumps from point \( E_0 \) to the point \( A \) where it joins the stable saddle path to capitalise on the higher stock returns. The higher stock prices at the point \( A \) trigger investment, which gradually lowers earnings per unit of capital and, therefore, stock prices. Final equilibrium is reached at the point \( E_1 \) where the capital stock has fully adjusted and stock prices are back to their initial equilibrium, where \( \alpha Y/K \) equals its initial level.
The intuition behind this result is as follows: Technological progress increases the marginal productivity of capital, therefore the value of equity per unit of capital and Tobin’s $q$. Stock prices jump because the stock market capitalises on the temporarily higher earnings. The higher stock prices, however, initiate a capital deepening process that gradually lowers the marginal productivity of capital and therefore revenue per unit of capital. The adjustment terminates when the initial $Y/K$-ratio and $Q$ are reached. The speed of adjustment towards $E_1$ depends most importantly on the investment adjustment costs.\footnote{In the model of Madsen and Davis (2006) technological revolutions lower stock prices of existing companies due to investment specific technological progress that lowers the effective acquisition costs of capital. In Schumpeterian growth model of Aghion and Howitt (1992) stock prices for existing companies decrease as a result of creative destruction following technological progress.}

The model also implies that stock prices are independent of changes in the required stock returns in the steady state. Shifts in required returns shift the $\dot{Q} = 0$ schedule along the horizontal $\dot{K} = 0$-curve and stock prices remain unaffected in the steady state. Intuitively, the reduction in the required returns increases $q$ and, therefore, initiates a capital accumulation process, which, due to diminishing returns to capital, lowers returns to stocks. The net investment terminates first when returns to stocks equal the lower required returns. Consequently, stock prices are only temporarily affected by the shift in required returns. This result departs from the conventional analysis that is based on dividend discount valuation models where stock prices are inversely related to required returns. However, standard valuation models do not account for the fact that, in the absence of taxes, capital investment endogenously responds to changes in the required stock returns until the ratio between the marginal productivity of capital and required returns equals one.

A change in taxes permanently influences stock prices on existing capital in the model because these changes shift the $\dot{K} = 0$ schedule and thus stock prices. However, this effect does not apply to claims on new capital stock. To appreciate this point consider an introduction of a tax credit
that lowers the effective acquisition cost of capital and increases the capital stock to a level where the reduction in the discounted stock returns equals the investment credit per unit of capital. This implies that net stock returns are unaltered for the companies that undertake the investment, whereas returns to the existing capital stock diminish, which, adversely affects the stocks of existing companies with low investment. In due course, however, when all capital stock has been replaced, stock prices revert to their initial level.

3.2 Capital market imperfections

An interesting special case arises under imperfections in the capital markets. The literature on imperfections in the capital market highlight cost of adverse selection and moral hazard (Hubbard, 1998). In the presence of incentive problems associated with costly monitoring of managerial actions, shareholders require higher returns to compensate for potential moral hazard. This is related to managers’ control over investment funds and higher monitoring costs (Bernanke, Gertler, and Gilchrist, 1996). Consequently, under the presence of imperfect information about the quality of a borrowers’ investment project, adverse selection leads to a wedge between external financing in an uninformed stock market and internally generated funds (Myers and Majluf, 1984).

The introduction of capital market imperfections into the framework here has two implications for stock markets. First, stockholders will require higher returns on their stocks compared to the event of perfect capital markets. This implies that the position of the $\bar{Q} = 0$ schedule is to the left of the position that would prevail under perfect capital markets. Since capital stock will endogenously adjust until the after-tax returns to capital equal the required returns, the price of stocks will be unaltered. However, stock returns are permanently higher.

Second, since the shadow cost of funds for investment projects need not carry monitoring costs, managers will have monetary incentives to finance projects from internal funds. In the presence of moral hazard and adverse selection the company has an incentive to use internal funds for investment funding, for which the shadow costs of internal financing is lower than that of external financing. As retained earnings increase the value of the firm, stock holders will experience a capital gain as compensation for the lower dividend payout. Since the dividend tax rate is typically higher than the capital gain tax rate, the incentive-induced financing of investment by retained earnings lowers the effective acquisition cost of capital. Consequently, resulting in a downward shift in the $\bar{K} = 0$ schedule. This result has important implications for stock prices and capital stock. It implies that the capital stock is higher and stock prices lower under imperfections in capital markets. More interestingly it yields pro-cyclical stock prices because the cost of moral hazard and adverse

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5 I am grateful to a referee for suggesting this subsection.
6 Summers (1981) show formally how to incorporate dividend and capital gain taxes into a Tobin’s $q$ model of investment.
selection varies inversely with pro-cyclical net-worth of the firms (see for instance Hubbard, 1998). Thus, the firm has a larger incentive to use internal funding for investment in periods of recessions than during upturns.

The effects of cyclicality in monitoring costs are shown in Figure 3. Suppose an unexpected recession that lowers profits leads to a left-ward shift the \( \hat{Q} = 0 \) schedule. Stock prices jump to the point A. The lower stock prices and, therefore, the lower value of the firm increases the required returns and shift the \( \hat{Q} = 0 \) schedule further to the left to \( \hat{Q}_2 = 0 \). The \( \hat{K} = 0 \) schedule shifts down because the higher monitoring costs have lead to an increasing reliance of investment on internal funds. This pushes investment past the point at which a dollar of retained earnings raises the market value by one dollar. In response to the shifts in the \( \hat{Q} = 0 \) and the \( \hat{K} = 0 \) schedules the stock market jumps to the point B. Thereafter, the economy moves up the new stable saddle path towards the new long-run equilibrium at \( E_1 \). The additional fall in stock prices from the point A to B that is induced by credit market imperfections can, to some extent, explain why stock prices are sensitive to cyclical fluctuations in earnings. This is particularly apparent when the fact that fluctuations in current profits are only a small fraction of discounted profits.

### 3.3 Imperfect competition

In the perfect competition case above, earnings per unit of capital is explained by the \( Y/K \) ratio. The \( Y/K \) ratio, however, is far too stable over time to explain the large historical fluctuations in real stock prices on medium term frequencies.\(^7\) The medium term movements in the profit rate, however, are highly correlated with factor shares. Factor shares exhibit large swings on medium term frequencies, as shown in the empirical section. The influence of factor share movements on stock prices and stock returns is incorporated into the model by relaxing the assumption of perfect competition and by introducing the labour market.

Under imperfect competition the total pre-tax real earnings in the economy are \( \Pi = S^K Y \), where \( S^K \) is capital’s income share. Thus the pre-tax earnings per unit of capital are equal to:

\[
\frac{\Pi}{K} = (1 - S^L) \frac{Y}{K},
\]

where \( S^L = WL/YP \) is labour’s pre-tax income share. Substituting this expression into Equation (2) yields:

\[
\hat{Q}(t) = (\rho + \delta)Q(t) - (1 - \tau(t))\frac{Y(t)}{K(t)} (1 - S^L(t)).
\] (8)

\(^7\) Under the CES technology assumption the \( Y/K \) ratio in the model above is replaced by \( (Y/K)^{1/\sigma} \), where \( \sigma \) is the elasticity of substitution. In the short run \( \sigma \) is likely to be less than one and the CES technology will consequently give rise to fluctuations in earnings that are somewhat larger than in the Cobb-Douglas case. However, the fluctuations in \( Y/K \), even for very small \( \sigma \), are not sufficient to generate the observed earnings.
This equation shows that expected stock returns equal capital gains plus earnings per unit of capital. Earnings per unit of capital are positive functions of the marginal productivity of capital and monopoly profits, where monopoly profits are inversely related to the share of income going to labour. Factor shares are determined in the labour market.

### 3.4 Determination of factor shares

To show the factors that determine $S^L$ consider the following standard augmented Phillips curve, which summarises the supply side of the economy:\(^8\)

\[
\dot{w}(t) = \beta_1 \pi^v(t) + \beta_2 MP_L(t) + \beta_3 (U(t) - \phi) + \dot{z}'(t) \beta_4,
\]  

(9)

where lowercase roman letters are logs of capital roman letters. Here, $\pi^v$ is the rate of inflation measured by the value-added price-deflator, $U$ is the rate of unemployment, $Z$ is a vector of wage push variables, $MP_L$ is the marginal productivity of labour, $\phi$ is the equilibrium rate of unemployment, and $\beta_1-\beta_4$ are constants. The equilibrium rate of unemployment is probably time-varying, but is set to a constant here for expositional simplicity. The $Z$-variables consist of direct and indirect taxes, relative commodity prices, relative food prices, union wage pushiness, relative minimum wages, unemployment benefits, mismatch, and other wage push variables (see Madsen, 1998).

By imposing long-run price and productivity homogeneity, $\beta_1 = \beta_2 = 1$, Equation (9) reduces to:\(^9\)

\[
\dot{s}^L(t) = -\beta_3 (U(t) - \phi) + \dot{z}'(t) \beta_4.
\]  

(10)

This equation shows that labour’s share of income is intimately related to the deviation of unemployment from its equilibrium and supply shocks. Unemployment above its equilibrium reduces the growth in wages until lower wage growth has brought unemployment back to its natural rate. Supply shocks change the wedge between labour costs and the value-added price-deflator and consequently influence firms’ earnings.

Since unemployment is an endogenous variable it needs to be solved out of Equation (10). This is done by letting unemployment be a function of factor shares (see for instance Bruno and

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\(^8\) The Phillips curve can easily be derived from an optimising framework (see for instance Mankiw and Reis, 2002). A complete model of the labour market is not presented here to keep the presentation as simple as possible.

\(^9\) These assumptions are standard in the natural rate of unemployment framework. If they are not satisfied then a unique equilibrium unemployment rate does not exist.
Substituting $U = a s^L$ into Equation (10) yields the following first-order difference equation:

$$s^L(t) = -\beta_3 (a s^L(t) - \phi) + Z'(t) \beta_4$$

(11)

This first-order difference equation has a stable equilibrium since $a$ and $\beta_3$ are both positive constants.\(^{11}\)

The labour market feeds into stock prices through the mark-up term in the stock valuation equation. Suppose that the economy is hit by an unanticipated adverse supply shock that brings wages in excess of the full employment equilibrium. The resulting increase in $s^L$ squeezes monopoly profits and hence earnings per unit of capital. Stock prices drop immediately due to the prospect of temporarily lower earnings. Since unemployment is in excess of its equilibrium labour’s share will be reduced until unemployment reverts to its natural level. Earnings per unit of capital and stock prices are back to their initial level when the labour market returns to its equilibrium. This adjustment path, however, only applies to the case where the capital stock is exogenous and hence unaffected by stock prices. When the capital stock is endogenous the adjustment path is quite different as shown next.

### 3.5 General equilibrium

Equations (6), (8), and (11) define the following simplified simultaneous linearized first-order differential system:

$$\dot{q}(t) = q(t) \rho - \alpha_1 [y(\omega) - k(t)] + \alpha_2 s^L(t)$$  

(12)

$$\dot{k}(t) = \beta (q(t) - \Gamma(t) - 1)$$  

(13)

$$\dot{s}^L(t) = -\theta s^L(t) + \varepsilon$$  

(14)

where $\alpha_1$, $\alpha_2$, $\theta$, and $\beta$ are positive constants, $\omega$ is technology shocks, and $\varepsilon$ is supply shocks. As notes above, lower case roman letters are logs of roman capital letters.

Linearizing the equation system around its steady state yields:

\(^{10}\) The unemployment rate is also a function of real wages in the model derived in Section 3 because employment is a negative function of real wages.\(^{11}\) Since $Z$ tends to zero in the long run, Equation (11) has the following solution:

$$s^L(t) = (\phi / a) + [s^L(0) - (1 + a \beta_3) \phi / a][(1 + a \beta_3)^{-t}$$

This equation converges to a constant steady-state equilibrium as follows:

$$\lim_{t \to \infty} s^L(t) = (1 + a \beta_3) \phi / a] = \bar{s}^L.$$
The system has two stable roots \((-\theta, \mu)\) because of its block recursive nature and because the system
given in Section 3.1, is stable (see the appendix).

As shown in the appendix the steady state multipliers for stock prices are given by:

\[
\frac{\partial q}{\partial \varepsilon} = \frac{\partial q}{\partial \omega} = \frac{\partial q}{\partial \rho} = 0,
\]

which shows that stock prices converge to a constant in the long run following technology shocks, supply shocks, and shifts in the required returns. The result that \(\frac{\partial q}{\partial \omega} = 0\) follows from the analysis in Section 3.1. An unexpected adverse supply shock leads to an instantaneous reduction in stock prices, which in turn lowers the capital stock until the pre-shock earnings per unit of capital are established. A shift in the required returns leads to an endogenous response in the capital stock as analysed above.

4 Implications of the model

The model has implications for the dynamics of stock returns following technology, supply and demand shocks, the movements of real stock prices in the long run and on medium frequencies, the equity puzzle, the effects of the ‘New Economy’ on stock prices, and the riskiness of stock investment for short and long horizon investors. These issues are now considered.

4.1 Growth in stock prices and dividends in the long run

The model predicts that real stock prices in the long run are mean reverting if all earnings are paid out because earnings per unit of capital revert toward the required returns plus depreciation. Allowing for retained earnings it follows that earnings per share and, therefore, real stock prices and dividends grow at the rate of \(rk\), where \(k\) is the retention ratio and \(r\) is the returns to new investment (see Copeland and Weston, 1992). Clearly, \(r\) may temporarily deviate from \(\rho\) following technology or supply shocks, however, \(r\) equals \(\rho\) in the steady state because of the endogenous response in the capital stock. Hence, the steady state growth in real stock prices is \(\rho k\), or about 3-4% annually depending on the exact magnitudes of \(\rho\) and \(k\). From this it follows that earnings and dividend growth rates are unrelated to the growth in GDP and labour productivity. This is because the retention ratio and the marginal productivity of labour are both unrelated to growth in GDP and

\[\text{Stability of the two-dimensional system follows automatically from Equation (15) by elimination of the labour market}\]

\[\text{since } -\alpha, \beta(1 - \partial y / \partial k) < 0.\]
labour productivity. In fact the marginal productivity of capital is constant along a balanced growth path in standard growth models.

4.2 The required returns
A recent controversy in finance is whether the high ex post equity premium experienced in the 20th century will 1) remain high into this century; 2) whether it has been permanently reduced in the post-war period; and 3), therefore, whether share holders can expect a substantial reduction in stock returns relative to bond returns in the future. Measurement problems, however, have rendered it difficult to assess expected stock returns and have led to results that are highly sensitive to the assumptions regarding the underlying process governing the expected growth in dividends (see, for different approaches and model assumptions, Arnott and Bernstein, 2002, Claus and Thomas, 2001, Fama and French, 2002, Harris and Marston, 2001). The model in this paper can be used to assess the required stock returns. Due to the endogenous response of the capital stock changes in the required returns can be directly read from the $Y/K$ ratio or from earnings per unit of capital.

A permanent shift in $\rho$ will permanently shift the $\dot{Q}=0$ curve and consequently alter the $Y/K$ ratio and, therefore, the returns to capital. All earnings shocks will be temporary because capital will adjust endogenously until the initial $Y/K$ ratio is established. Changes in the tax structure can also change the $Y/K$ ratio but not the after tax returns to capital. An investment credit, for instance, will increase the capital stock until the initial after-tax capital returns are established.

More explicitly the steady state value of the required returns is given by the solution to Equations (6) and (8), in steady state this is given by:

$$\rho = \left( \frac{1-\tau}{1-\Gamma} \right) \left( \frac{\bar{Y}}{K} \right) \left( 1-\bar{S}^\tau \right) - \delta = \left( \frac{1-\tau}{1-\Gamma} \right) \left( \frac{\bar{Y}}{K} \right) \left( 1-(1+a\beta_3)(\alpha + \phi/a) \right) - \delta .$$

This equation shows that the required stock returns are reflected in the steady state $Y/K$ ratio provided that the tax structure and the equilibrium rate of unemployment remain unaltered. To simplify the model consider the perfect competition counterpart of the required returns.

$$\rho = \alpha \left( \frac{1-\tau}{1-\Gamma} \right) \left( \frac{\bar{Y}}{K} \right) - \delta$$

(16)

For the US $(1-\tau)/(1-\Gamma)$ is close to one and shows little variation over time, as shown by the estimates of Summers (1981). The use of the $Y/K$ ratio as an approximation for $\bar{Y}/\bar{K}$ hinges on an assumption of low adjustment costs of investment. Cummins et al. (1996), for example, find

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13 Growth in real share prices and dividends are often related to growth in GDP and labour productivity. The problem associated with this approach is that economic growth is usually associated with capital deepening, which implies that earnings are dispersed over more units of capital. See Davis and Madsen (2008) for an exposition.
investment adjustment costs to be well below 10% of the investment acquisition costs for most OECD countries. This suggests low costs of adjusting the capital stock to its desired level.

From Equation (16) it follows that earnings per unit of capital are mean-reverting. Equation (16) shows that, under perfect competition, after-tax earnings per unit of capital converge to a constant, equal to an exogenous given required returns plus the depreciation rate. The intuition behind this result is that capital stock adjusts endogenously to the tax-adjusted required returns. Note that Equation (16), in the absence of taxes, differs from Equation (3) in that in Equation (3) the required returns equal earnings plus capital gain, capital gains are absent from Equation (16). This is because stock prices are constant in steady state and capital gain is, therefore, zero.

Note that Equation (16) does not indicate a causal relationship. For a given required stock return, changes in the parameters on the right-hand-side will be counterbalanced by changes in the $Y/K$ ratio. Changes in the $Y/K$ ratio need, therefore, not reflect changes in $\rho$ but could be an endogenous response to changes in the equilibrium unemployment rate or taxes. In the absence of changes in the tax structure and the equilibrium unemployment rate, however, changes in the $Y/K$ ratio will be a good measure of changes in the required stock returns.

The path of the $Y/K$ ratio over the past 100 years for 13 OECD countries is displayed in Figure 4, where $K$ is based on the perpetual inventory method for total fixed investment. Since the curve exhibits low variability, the $Y/K$ trend is easily identifiable. The $Y/K$ ratio only deviates significantly from its long run trend in periods of major disruptions, such as the Great Depression and WWII. The $Y/K$ ratio fluctuated around a constant slightly below 0.8 before 1950, declined thereafter and has stabilised slightly above 0.5 during the past three decades. The 2.5-percentage point decline in the $Y/K$ ratio corresponds to a 32% reduction in the required returns. This result is consistent with the


---

Data from the following 13 countries are included in the figure: Canada, the US, Australia, Denmark, Finland, France, Germany, Italy, the Netherlands, Norway, Spain, Sweden, and the UK. See Madsen (2003) for data sources.
result of Fama and French (2002) for the US and it is interesting to note that they use 1950 as the benchmark year of the decline in the required returns, which corresponds to the results obtained above.

As an alternative to the \( Y/K \) ratio Figure 5 displays earnings per unit of capital adjusted for the influence of the business cycle and the deviation of factor shares from their long-term equilibrium for the same countries as those included in Figure 1.\(^{15}\) Like the \( Y/K \) ratio, the figure indicates that earnings per unit of capital have stabilised at a lower level in the post WWII-war period. In comparison with the pre WWI period earnings per unit of capital have been reduced by about 3 percentage points or 38%. These results are consistent with the results from the \( Y/K \) ratio and suggest that the required stock returns, and hence the expected returns to stocks, have been permanently reduced to a lower level.

4.3 Stock prices and technology innovations
The stock market run-up in the 1990s has often been attributed the technology innovations that were derived from the ‘New Economy’. The model in this paper implies that technological innovations only have short-term effects on real stock prices and dividends as analysed in Section 3. This result is in line with the result of Datta and Dixon (2002). They show that technological innovations only have temporary effects on monopoly profits, as the entry of new firms drives the extraordinary profits down to zero in the medium term.

\(^{15}\) The following procedure is used to cyclically adjust earnings per unit of capital. Earnings per unit of capital for each individual country are regressed on a constant, the deviation of the log of manufacturing factor shares and the deviation of the log of real GDP per capita from their time-trends. The estimated coefficients of factor shares and the business cycle times the cyclical factor shares and cyclical per capita income, respectively, are then subtracted from earnings per unit of capital.
To show the implications of technology innovations on stock returns more explicitly, the marginal productivity of capital under the Cobb-Douglas technology assumption can be written as:

$$\Delta \ln \left( \frac{Y}{K} \right) = \Delta \ln A + (\alpha - 1) \ln \left( \frac{K}{L} \right),$$

which shows that the growth in the marginal productivity of capital depends on two counterbalancing forces, namely technological progress and capital deepening. From this equation it follows that technological progress increases the marginal productivity of capital, which leads to higher stock prices. The higher stock prices, in turn, initiates capital deepening that continues until the marginal productivity of capital is back to its initial equilibrium. Historically, capital deepening multiplied by capital’s share of income has grown at approximately the rate of technological progress, in the long run. This can be seen from Figure 4 and has prevented the marginal productivity of capital from growing.

4.4 Stocks and long-horizon investment

Traditional static financial theory of optimal portfolio allocation has concentrated on the mean and variance of asset returns without paying much attention to intertemporal decision-making. In the seminal paper of Samuelson (1969) it is shown that if asset returns are stochastic then the optimal portfolio of an investor with a power utility function is independent of the investment horizon. One important assumption underlying Samuelson’s model is that the accumulated stock index is assumed to follow a random walk and therefore that stock returns are independently distributed over time. Random walks have the important properties that the variance is independent of the data frequency. If risk is measured as the variance of the asset’s return or the covariance between returns and consumption growth, the riskiness of the asset is independent of the investment horizon.

This paper has shown that the log of real stock prices and stock returns are mean reverting and, therefore, do not follow a random walk. The mean-reverting property implies that the riskiness of investment is a declining function of the investment horizon. Thus, stocks should have relatively higher weights in long horizon portfolios than in short-term portfolios for the risk-averse investor.

4.5 Valuation models

The analysis in Section 3.1 suggests that real stock prices grow at the rate of $\rho \kappa$ in the steady state, which implies that that static valuation models cannot be used as tools for stock price valuation. A problem that is associated with the static valuation models is the factors that determine stock prices are endogenous, thus, rendering many standard valuation models less useful tools for stock price valuation.
To see the consequences of the endogeneity problem in valuation models consider Gordon’s growth model:

\[ Q_t = \frac{D_{t+1}^e}{\rho - g} = \frac{D_{t+1}^e}{\rho - r \kappa}, \]

where \( D_{t+1}^e \) is expected dividends per share at period \( t+1 \), conditional on all available information at period \( t \), \( r \) is the permanent returns on new investment and \( g \) is the expected growth rate in dividends per share.

Provided that the right-hand side variables in Gordon’s growth model are exogenous, the effects on stock prices of shifts in \( \rho \), \( g \) and \( D \) can be readily computed. However, in contrast to this common assumption, these variables are not exogenous but interdependent. First, since earnings per unit of capital converge to a constant for a given \( \rho \), as shown in the previous section, it follows that the returns on new investment, \( r \), cannot permanently deviate from \( \rho \). The denominator in Equation (17) consequently collapses to \( \rho (1 - \kappa) \). Hence, \( g \) will always be constant in Gordon’s model unless the retention ratio permanently changes. However, since changes in the numerator and denominator following a change in the retention ratio are the same, the valuation is independent of the retention ratio since taxes are absent from this valuation model.

Second, stock prices are independent of \( \rho \) as shown in the previous section. A shift in \( \rho \) changes the steady-state dividends per share correspondingly. Consider a reduction in \( \rho \). From a steady state equilibrium where the denominator in Equation (17) is given by \( \rho (1 - \kappa) \), a reduction in \( \rho \) to half its value, for instance, results in a 100% increase in stock prices if dividends are assumed to be exogenous. The model in this paper suggests that dividends in steady state will be reduced by the same percentage as the percentage reduction in \( \rho \) and stock prices will consequently only be temporarily affected by the change due to adjustment costs in investment and employment. Finally, innovations in dividends that are not caused by changes in \( \rho \) or taxes, will not have permanent effects on dividends as shown above because the capital stock endogenously adjusts until dividends equal \( \rho (1 - \kappa) \).

It is, therefore, a straightforward task to find the fundamental value of stocks in the steady state by using the recursive formula \( Q_{t+1} = Q_t (1 + \rho \kappa) \), where \( \rho \) is given by Equation (16) or \( \rho \kappa \) is assumed to be constant or is allowed to evolve slowly. Since earnings are constantly changing due to shocks in demand, supply and technology, a more dynamic version of the Gordon growth model that takes these temporary shocks into account, such as the Campbell-Shiller model, will be needed. The initial jump in stock prices following a technology shock, \( \omega \), can then be computed from the following equation as derived in the appendix:
where $\mu$ is a stable root.

However, this approach has practical problems in that it is too complex and cumbersome. A simple approximation is to use the following approximation equation, which is derived in Copeland and Weston (1992):

$$Q_t \approx \frac{D^*_{t+1}[1+\kappa \cdot T(r-\rho)]}{\rho(1-\kappa)}, \quad (18)$$

where $T$ is number of years for which the extraordinary profits last. Using the result that $Q_{t+1} = Q_t(1 + \rho \kappa)$ in steady state, Equation (18) collapses to:

$$Q_t \approx [1+\kappa \cdot T(r-\rho)][1+\rho \kappa]Q_{t-1}, \quad (19)$$

where $r$ and $T$ are allowed to change to reflect the fact that earnings are constantly exposed to shocks. From Equation (19) we get the following approximation

$$\frac{dQ/Q}{dr} \approx 0.5T,$$

where $(1 + \rho \kappa)\kappa$ is set to 0.5, which is close to its historical average. This equation shows the percentage change in stock prices in response to a one percentage point excess return that lasts for $T$ periods. Figure 1 shows that earnings per unit of capital are rarely more than two percentage points in excess of their long run equilibrium, for more than five years. This limit translates to a stock prices increase of 5%. Hence, stock prices ought to be relatively insensitive to earnings shocks, unless required stock returns fluctuate substantially over the business cycle as a result of imperfect credit markets.

The steady-state P/E ratio is $\rho^{-1}$. If there is a temporary earning shocks the P/E ratio is given by:

$$\frac{P_t}{E_t} \approx \frac{1+\kappa \cdot T(r-\rho)}{\rho},$$

where $E$ is earnings per share or earnings per unit of capital under certain regularity conditions. In this paper it has been argued that trend real earnings is the essential measure in the price-earnings ratio because real earnings converge towards a constant growth trend which is given by $E_{t+1} = E_t(1 + \rho \kappa)$. 

\[ \hat{q}_{t=0} = \frac{\partial y / \partial \omega}{\beta(1 - \partial y / \partial k)^2} \mu^2 d\omega \]
5 Empirical estimates

The most important empirical implications of the model of this paper are 1) that factor shares are important determinants of stock prices; 2) that stock prices are negatively affected by the output-capital ratio because the output-capital ratio echoes $\rho$; and 3) that real stock prices are trend stationary in the sense that they fluctuate about increasing trends provided that $\rho \kappa$ is approximately constant. These implications are examined using pooled cross-section and time-series data for the OECD countries.

To investigate these issues, the following stochastic counterpart of Equation (8) is estimated for the 21 OECD countries that are listed in the notes to Table 1, using panel data over the period from 1953 to 2001:

$$\Delta \ln Q_{it} = a_0 + a_1 \Delta \ln \left( \frac{Y}{K} \right)_{it} + a_2 \Delta \ln S^K_{it} + a_3 \Delta \ln S^{Ke}_{t+1} + a_4 \Delta \tau_{it} + a_5 \Delta \ln Y_{t+1}^{e} + a_6 \Delta \ln \left( \frac{S^K_{t+1}}{S^K_{t}} \right) + a_7 \ln \left( Q_{t-1} / \bar{Q}_t \right) + v_{it},$$

where the subscript $i$ signifies country $i$, $v$ is a stochastic error term, $Q$ is stock prices deflated by consumer prices, $\tau$ is corporate taxes divided by accounting profits before tax, $Y/K$ is GNP divided by non-residential capital stock, the superscript $e$ refers to expected value, and $S^K$ is the share of capital in total income and is measured as net operating surplus divided by nominal GNP. Expected income growth is included in the estimation equation to allow for pro-cyclicality of earnings. Stock price indices with the broadest sectoral coverage are used. Data sources are detailed in Madsen (2003). All the regressors are instrumented as detailed in the notes to Table 1.

The variable $(Q_{t-1} / \bar{Q}_t)$ is an error-correction term and is denoted $ECT$ in Table 1 below. It ensures that real stock prices converge toward their long-run equilibrium, which is defined here as a deterministic trend; thus implicitly imposing the restriction that $\rho \kappa$ is constant for each individual country in the estimation period. More precisely, $\ln(Q_{t-1} / \bar{Q}_t)$ is the lagged residuals from regressing the log of real stock prices on a time trend for each individual country over the period from 1953 to 2001. A similar method is used to compute $(S^K_{t-1} / \bar{S}_t^{K})$. This term is included in the model to allow for adjustment of real stock prices to disequilibria in the factor market following the dynamic path in the phase diagram (Figure 2) above. The coefficient of this term is negative in a perfect foresight stock market but positive in a stock market that extrapolates the trend in earnings and, as such, ignores the mean-reverting nature of factor shares. Suppose that capital’s share in total income is above its steady state equilibrium. The perfect foresight stock market is aware of the fact that capital’s share is above its steady state equilibrium.

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16 Corporate tax rates are calculated as total corporate taxes divided by net operating surplus. Net operating surplus is from OECD, National Accounts, and corporate taxes are from UN, Yearbook of National Accounts Statistics and OECD, Revenue Statistics of OECD Member Countries.
that earnings will subsequently be reduced to their steady state level and stock prices will consequently adversely respond to the disequilibrium situation.

To gain efficiency and to correct for serial correlation and heteroscedasticity, the covariance matrix is weighted by the correlation of the disturbance terms using the variance-covariance structure as follows:

\[ E\{\varepsilon_{it}^2\} = \sigma_i^2, \quad i = 1, 2, \ldots, N, \]
\[ E\{\varepsilon_{it}, \varepsilon_{jt}\} = \sigma_{ij}, \quad i \neq j, \]
\[ \varepsilon_{it} = \rho \varepsilon_{it-1} + \nu_{it} \]

where \( \sigma_i^2 \) is the variance of the disturbance terms for country \( i = 1, 2, \ldots, N \), \( \sigma_{ij} \) is the covariance of the disturbance terms across countries \( i \) and \( j \), \( \varepsilon \) is the disturbance term and \( \nu \) is an \textit{iid} disturbance term. The variance \( \sigma_i^2 \) is assumed to be constant over time but to vary across countries. The error terms are assumed to be mutually correlated across countries, \( \sigma_{ij} \), as random shocks are likely to impact all countries at the same time. The parameters \( \sigma_i^2 \), \( \rho \) and \( \sigma_{ij} \) are estimated using feasible generalized least squares.

Table 1. Parameter estimates of Equation (20).

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>( R^2 )</th>
<th>( DW )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \ln(Y/K) )</td>
<td>-2.93(4.56)</td>
<td></td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td>( \Delta \ln S_t^K )</td>
<td>0.70(2.18)</td>
<td>( \ln(S_{t-1}^K / \overline{S}_{t-1}^K) )</td>
<td>1.06(7.97)</td>
<td>1.84</td>
</tr>
<tr>
<td>( \Delta \ln S_t^{K, e} )</td>
<td>5.19(6.01)</td>
<td>( ECT )</td>
<td>-0.20(10.0)</td>
<td></td>
</tr>
<tr>
<td>( \Delta \tau_t )</td>
<td>-0.31(1.13)</td>
<td>Constant</td>
<td>-0.13(6.16)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Absolute \( t \)-statistics are given in the parentheses. \( DW \) = Durbin-Watson test for first order serial correlation in fixed effects panel data models. \( ECT \) = error-correction term. The following variables in first differences are used as instruments for the right-hand-side variables (except for \( ECT \) and \( \ln(S_{t-1}^K / \overline{S}_{t-1}^K) \)):

Lagged dependent variable, one period lag of the real interest rate, unlagged values and one period lag of the log of real GNP, a one period lag of the log of real stock prices, and the log of consumer prices. The following 21 countries are included in the data sample: Canada, USA, Japan, Australia, New Zealand, Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the UK. Estimation period: 1955 to 2000.

The results of estimating Equation (20) are presented in Table 1. The estimated coefficients of capital’s income share are economically and statistically highly significant. Since \( S^K \) typically fluctuates by 10-25% on medium term frequencies it is highly influential for the medium-term fluctuations in real stock prices. For example, \( S^K \) increased by approximately 20% from the mid 1970s to 2000 and accounts for an almost 120% increase in real stock prices over the same period. This not only suggests that fluctuations in factors shares are very influential for movements in stock prices. It also indicates that stock markets overreact to changes in factor shares and ignore that factor...
shares will revert to a constant in the long run given that the model in this paper predicts that the real stock price elasticity of capital’s income share is significantly less than one.

To put this result into context of the Gordon growth model consider an increase in income shares that is expected to be permanent. If the expected growth in earnings is expected to be unaffected by the increasing earnings Gordon’s growth model predicts a coefficient of capital’s income share of one. Since the sum of the estimated coefficients of capital’s income share in Table 1 is 5.89 it follows that a large fraction of the innovation in factor shares must be built into the expected growth term in the Gordon model. This result suggests that stock markets overreact to innovations in earnings in two respects: First, they fail to acknowledge that factor shares converge to constants in the long run. Second, level changes in earnings build into growth expectations although earning growth will be revised in the future so that earnings per unit of capital remain constant.

The estimated coefficient of $Y/K$ is statistically and economically significant and has the expected negative sign. This result highlights the analytical findings in the previous section that the required stock returns move proportionally to the movements in $Y/K$ and, therefore, that real stock prices are inversely related to $Y/K$. The estimated coefficient of expected income growth is economically and statistically highly significant, which suggests that stock prices are very sensitive to business cycle fluctuations. Finally, the estimated coefficient of the error-correction term has the expected negative sign and is statistically highly significant, which suggests that real stock prices are mean-reverting around an upward trend.

Overall, the estimates are consistent with the predictions of the model presented in the previous section. First, factor shares are key determinants of share prices, suggesting that the labour market plays a central role for stock prices and ex post stock returns. It is noteworthy that strikes per worker are increasing in periods of reduced corporate earnings, such as the mid 1970s, and are decreasing in periods of increasing earnings per unit of capital, such as the past two decades. Second, real stock prices tend to gravitate toward a mean that is increasing over time because earnings are retained within the company. Third, stock prices are negatively related to $Y/K$ because $Y/K$ mirrors the required returns. For example, an increase in the required returns, for instance, lowers stock prices and initiates a capital reducing process which increases $Y/K$ due to diminishing returns to capital.

6 Summary and conclusion

This paper has presented a theoretical framework that explains the long run behaviour of stocks and dividends. The model predicts that earnings per unit of capital fluctuate on medium term frequencies due to innovations in earnings, in particular, and required returns, but converge towards a constant mean in the long run due to a Tobin’s $q$ effect. This implies that real stock prices and dividends are
not growing over time, in their steady state, due to increasing earnings per unit of capital but solely because earnings are retained within the company. Furthermore, shifts in earnings or required stock returns have no permanent effects on real stock prices and dividends. Hence, standard valuation model such as the Gordon growth model and other static valuation models, are rendered invalid for stock valuation. Real stock prices will only increase by the retention ratio multiplied by the required returns in the steady state equilibrium. More dynamic valuation models that allow for temporarily higher earnings are required for stock valuation.

The model also allows the required stock returns to be recovered from macroeconomic aggregates such as the output-capital ratio and adjusted earnings per unit of capital. The estimates in this paper suggest that the required stock returns have decreased by about 30% over the past century. Despite this reduction, however, stock returns can be expected to remain relatively high in the future. Coupled with the possibility of continuing low real interest rates the equity risk premium will remain high and, perhaps, still be a puzzle.

**APPENDIX**

The equation system given by Equation (15) has two stable roots \((-\theta, \mu)\) and one unstable root \((\xi)\). Assuming that \(-\theta \neq \mu\) the general solution to the system is:

\[
\begin{bmatrix}
q - \bar{q} \\
k - \bar{k} \\
S^L - \bar{S}^L
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 1 & 1 \\
2 & 2 & 2 & 2 \\
3 & 3 & 3 & 3
\end{bmatrix}
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\alpha_3
\end{bmatrix},
\]

where \(x^j_i\) are eigenvalues corresponding to the \(x^j\) eigenvectors. Deriving the eigenvectors from this system we get, after a few manipulations:

\[
\begin{align*}
q_i - \bar{q} &= -c_1 \theta e^{-\alpha_1} + c_2 \mu e^{\alpha_1} \\
k_i - \bar{k} &= c_1 \beta e^{-\alpha_1} + c_2 \beta e^{\alpha_1} \\
S^L - \bar{S}^L &= c_1 \frac{\mu + \xi + \theta}{\alpha_2} \theta + \xi \mu e^{-\alpha_1}.
\end{align*}
\]

Since \(c_1\) and \(c_2\) are determined by the initial conditions for \(k\) and \(S^L\) we need to solve the system at \(t_0\).

**Steady state multipliers**

Totally differentiating the system given by Equations (12)-(14), where \(\dot{q_i} = \dot{k_i} = \dot{S}^L = 0\), yields the following system:

\[
\begin{bmatrix}
\rho & \alpha_1 (1 - \dot{\gamma} / \dot{k}) & \alpha_2 \\
\beta & 0 & 0 \\
0 & 0 & -\theta
\end{bmatrix} \begin{bmatrix}
\dot{q} \\
\dot{k} \\
\dot{S}^L
\end{bmatrix} =
\begin{bmatrix}
\alpha_1 \dot{\gamma} / \dot{\omega} & 0 & 0 & -q \\
0 & 0 \beta & 0 \\
0 & -1 & 0 & 0
\end{bmatrix} \begin{bmatrix}
d\omega \\
d\epsilon \\
d\Gamma \\
d\rho
\end{bmatrix}.
\]
From this system the steady state stock market multipliers are given by:

\[
\frac{\partial q}{\partial \epsilon} = \frac{\partial q}{\partial \omega} = \frac{\partial q}{\partial \rho} = 0 \quad \text{and} \quad \frac{\partial q}{\partial \Gamma} = -\frac{1}{\alpha_i} < 0.
\]

From these multipliers it follows that stock prices in their steady state are unaffected by supply shocks, technology shocks and changes in the required returns. An increase in investment tax credits reduces stock prices by the inverse marginal productivity of capital because it lowers the effective acquisition cost of capital.

**Supply shock**

From the steady state multipliers the dynamic movements in the system following a technology shock are given by:

\[
q - \bar{q} = \frac{\partial y / \partial \omega}{\beta(1 - \partial y / \partial k)} e^\omega d\omega
\]

\[
k - \bar{k} = \frac{\partial y / \partial \omega}{(1 - \partial y / \partial k)} e^\omega d\omega
\]

\[
S^k - \bar{S}^k = 0.
\]

**Technology shock**

The dynamics of the system following a supply shock are given by:

\[
q - \bar{q} = 0 \cdot d\epsilon = 0
\]

\[
k - \bar{k} = -\frac{\alpha_2}{\alpha_1(1 - \partial y / \partial k)} d\epsilon
\]

\[
S^k - \bar{S}^k = \frac{1}{\theta} d\epsilon.
\]

**DATA APPENDIX**


REFERENCES


