

## Quality & Reliability Engineering (215.321)

### Test, 2009-02 – Open book/notes/internet

You have 50 minutes to complete the test. Show all work.

Value: 15% of final grade

1. When Cinderella's 2 cruel stepsisters arrive at the ball at 8pm, they discover that there is now a total of 100 maidens at the ball. The ball ends, of course, at midnight.

- a. If the prince dances with 10 maidens per hour, choosing at random from the 100 maidens each dance, what is the likelihood that he dances with at least one woman more than once? [3 marks]

It's far easier to calculate the reverse – no duplicate partners – then subtract that from 1. The no-dups case is  $100/100 \times 99/100 \times \dots \times 61/100$ . Short form:

$$\left(1 - \frac{(100)_{40}}{100^{40}}\right)$$

- b. What is the likelihood that he dances with at least one of the stepsisters before midnight? [3 marks]

Again, easier to calculate the reverse, which is  $(1 - (98/100)^{40})$

2. The fairy godmother wasn't completely straight with Cinderella – it seems that the magic she used wasn't really guaranteed to last until midnight. In fact, the coach had a 0.02 chance per hour of turning back into a pumpkin, and each of the four horses had a 0.01 chance per hour of turning back into a mouse. Even Cindy's gown had a 3% per hour chance of a wardrobe malfunction.

- a. Assuming she needs the coach, the gown, and three of the four horses to get home safe, what is the likelihood Cindy makes it until midnight? [3 marks]

Coach and gown are in series, just add the lambdas and multiply by 4 (hours) to get  $R(\text{coach\&gown})$ . Multiply that by  $R(\text{horses}) = R(4 \text{ horses}) + R(3 \text{ horses})$ .  $R(4 \text{ horses})$  is just  $(R(\text{horse}))^4$ , and  $R(3 \text{ horses}) = 4(R(\text{horse}))^3 \times (1 - R(\text{horse}))$ .

- b. It turns out that there was, in the boot of the coach, a spare mouse and emergency spell to turn it into a horse. What is the reliability of this system on its four hour mission? [3 marks]

The coach and gown calculation doesn't change.  $R(\text{horses})$  now has three terms, the two from part (a) and a third term which is given by the convolution of a two-horse failure by time  $\tau$  and three-horse no-

fail from  $\tau$  to four hours, i.e. this looks like a two-horse failure followed by a (presumed perfect) switching to a spare.

- c. Apparently magic glass slippers are a bit more durable (I always wondered about that...). The one the prince found has a hazard rate of 4% per day. Assuming it takes the prince a week to find Cinderella, what is the likelihood the slipper will last long enough? [2 marks]

This is just  $e^{(-0.04*7)}$ .

- d. What is the mean time to failure of a glass slipper? [1 marks]

MTTF =  $1/\lambda = 25$  days.