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Multi-Party and Spatial Influence Effects in Opinion Formation Models

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Abstract

Opinion formation amongst large groups of connected individual autonomous agents or voters is a complex phenomena to accurately model and indeed understand. The Sznajd model describes a simple set of rules for an individual voter to influence its neighbours and for an initially randomly-mixed set of opinion holders to converge to a single consensus opinion. We consider quantitative effects of having more than the usual two opposing opinions (or political parties) present and study what happens to the time to achieve consensus when many minority opinions might initially be present in a system. We report on numerical experiments with systems of up to ten opposing different opinions or political parties present in the original mix and also on the effect of increasing the number of neighbours a single voter can influence.

Keywords: policy issues; political process modelling; agent based modelling; opinion formation model; interdisciplinary modelling; Sznajd model.

1 Introduction

Sociological and other interdisciplinary models such as Sznajd's model of opinion formation [1] display some interesting phase transitional and complex behaviours. Typically these models cannot be explored analytically except through the simplest mean field approximations and therefore rely upon computer simulation for a more complete study.

Opinion formation models can capture individual behaviour at a microscopic level that manifests itself as a macroscopic or system-wide outcome when implemented with system of many participating agents. The Sznajd model of opinion formation exhibits complex phase transitional and growth behaviour and can be studied with numerical simulations on a number of different network structures [2]. In this present paper we consider nearest neighbour (distance of one cell) and next-nearest neighbour (distance of two cells) on square meshes. While this is not particularly realistic for real voter populations, it suffices to generate manageably large voter agent systems and facilitates study of relatively large systems sizes.

The Sznajd model is constructed as follows:

1. each cell on the mesh or lattice represents a voter agent holding a single opinion
2. starting conditions are chosen for a random mixture of Q different opinion states. Typically Q is 2 but in this paper we explore the effect of Q up to 10.
3. each of the N voters is considered (in a random sequence to avoid correlation sweeping effects)
4. upon choosing a voter, we randomly look at one of its neighbours.
5. if the voter and its neighbour hold the same opinion we set the opinions of all their immediate neighbours to this opinion
6. we repeat this process until consensus is reached (whereby all voters hold the same opinion state, whatever it might be)

There are a number of variations to the simple Sznajd model that have been explored in the literature. The notion that two voters with the same opinion will have a strong influence on their immediate neighbours is not unreasonable, although it is possible to vary the number of agreeing voters required to cause a local sway of opinion. Triples and plaquettes of four have been tried. It appears that a pair is enough to capture the essential model behaviour however, and we use the simpler "voter pairs" in this paper.

A simple regular mesh is not particularly realistic and a number of other meshes and graph networks including preferential and scale free network structures have also been studied and reported in the literature. Again, it appears that the square mesh captures the essence of the model behaviour and we use a simple square mesh in the work reported here. However we do consider the neighbourhood of influence of the agreeing pair of voters. We consider nearest-only and nearest-and-next-nearest neighbour combinations.

The Sznajd model and models like it can be run to full consensus under many dynamical conditions [3] . The square (two-dimensional) meshes we study do lead (eventually) to a consensus outcome. The consensus state is a stable state since there are no thermal or spontaneous changes of opinion in the model we study. However although the consensus state will be arrived at eventually in finite time for a finite system, these completion times do grow with system size. We explore this effect and the implications for the maximum feasible system sizes we can study.

The typical Sznajd model is described and illustrated in Section 2. In Section 3 we discuss the scalability of a computer simulation of the Sznajd model with large numbers of possible opinions and different neighbourhood influence distances. We present some results of these experiments in Section 4 showing the shape and shifts in the distribution in times-to-consensus exhibited by the model. We offer some explanatory discussion of these in Section 5 and some conclusions and ideas for further work in Section 6

2 Sznajd Opinion Formation Models

The Sznajd model has some features in common with models in statistical mechanics and physical systems [4] such as the propagation and percolation of opinions amongst agent voters [5]. One key difference from other statistical models such as the Ising system, is that the Sznajd model propagates information outwards from a individual cell, rather than using an inward merging operation [6]. It has been compared to game theoretical models [7] and has a number of specific applications [8] but the most interesting is that of modelling opinion formation amongst a network of agent voters.

The Sznajd model is not the only simple model of voting systems, organizational systems [9, 10] or opinion formation [11, 12]. Other models include the voter model [13]; various exclusion process models [14, 15] and the relative agreement model [16]. The Sznajd model does however have the advantage of being straightforward to program and allows relatively large voting agent systems to be simulated. Since many of the inferences that can be drawn from such models are statistical and depend on averaging rather than on individual absolute values, speed and ease of simulation is important to allow sampling of many different starting conditions without appreciable bias.

The Sznajd model can be generalised in a number of ways [17] and can be studied on different network an lattice topologies [18] but in this present paper we focus on simply increasing the number of possible starting opinions in a model system.

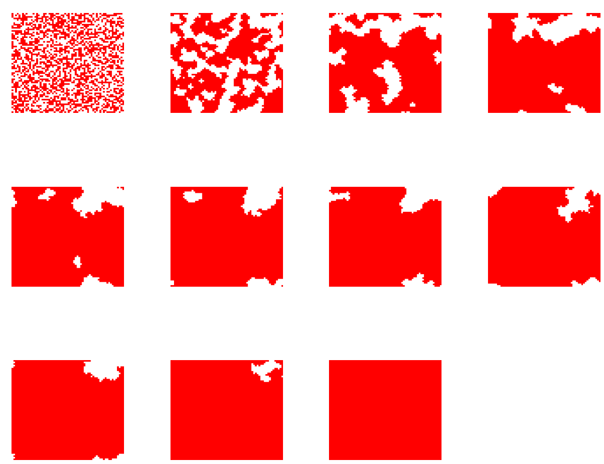

Figure 1: Sznajd Model on 64×64 periodic lattice at times: 0,9, 19, 29,...,79, 89 steps after random 50/50 start.

Figure 1 is perhaps the easiest way to explain how the model works. A series of screen-shots show a square lattice with periodic boundaries so that each voter cell has exactly the same number of neighbours - even at the edges. The Sznajd model system is initialised with a 50/50 random mix of two opinions held by the agents. The model is evolved according to the rules listed above and clusters of consolidated opinions start to form and grow, finally resulting in consensus when a single opinion dominates the entire system. A model time step consists of each site or voter having been "hit" once according to the procedure above, although the hits are in a random order to avoid sweeping correlation artifacts.

Real models of political voting system need not of course run to full consensus and it might suffice for an opinion or political party to "win" if it at some point reaches a critical majority such as 75% for example. The model is simpler to study and compare with other results in the literature however if we focus on the simulation time required to achieve full consensus.

Any particular system may exhibit a great number of fluctuations and an opinion that at one stage appears to be losing can quite easily reverse its fortunes and finally emerge as the winner. A theoretical analysis of the distribution of fluctuations and their sizes is beyond the scope of the present paper, but generally speaking the larger the model system, the richer the fluctuational possibilities. In this paper we consider what happens when much more than two initial opinions (or political parties) are present in the initial system. This has a significant effect on the time for the model system to achieve consensus.

3 Scalability of Large Systems

To simulate a large enough model system to capture the interesting behaviours, it was necessary to develop a custom simulation code. A custom C++ simulation code was specially written for the work reported here. The program code is straightforward enough, using an array of bytes to hold the opinions and using simple square geometry periodic boundary conditions. Different values of Q are supported as are different combination of nearest and next-nearest neighbour voter influences. The code initialises the system according to the number of opinions Q and automatically runs over 1,000 independent different starting configurations as discussed below. Consensus-achieving completion times for the model are recorded as a means with standard deviations or as log-2 binned histograms as presented in the section below. Random start conditions are generated with a quality random number generator with period approximately 10^{57}. Random deviates are also drawn to choose which sites to hit and which neighbours of a given site to compare. It is likely we have erred on the side of caution and a faster random number generator could be used. We note no detectable difference in the results presented by using different random number generator algorithms.

As Figure 2 shows, the computational time taken to perform a simulation of a randomly initialised Sznajd system on our square mesh rises faster than linearly with system length L. A system of around $64 \times 64 =$

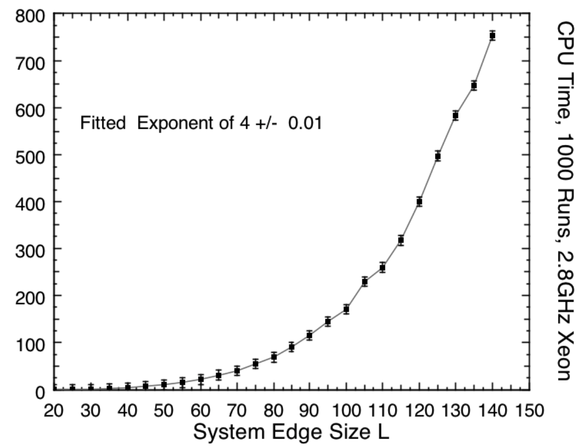

Figure 2: Processing time on 2.8GHz Intel Xeon core for different system sizes. A least-squares fit indicates $t \approx L^4$ or $t \approx N^2$

4096 cells is feasible for the statistical accuracy regime we a re interested in for the work reported in this paper. For timing purposes we have investigated system sizes up to 150×150 which can feasibility be run individually for visualisation purposes but not feasibly to consensus completion in any useful number of batch runs. A parallel computing engine could of course be used to obtain better job throughput and experimental statistics should it be required. However, in the work reported in this paper we do not believe there are multi scale fluctuations that would necessitate investigation with larger system sizes.

Generally simulations of this model are compute bound and not memory bound, although it is advantageous to have a compact, efficient and portable C++ implementation. Further optimisations to the simulation are possible but are largely only $O(N)$ and therefore of limited value when the phenomena itself appears to scale worse than this.

4 Results

The results reported below have been averaged over batches of 1000 runs each with different starting configurations. The nature of the Poissonian distributions with long tails make it difficult to obtain sensible uncertainty estimate s from simple standard deviation calculations. Consequently the error estimates shown on the plots in Figure 3 are obtained by running 10 different independent sets of the 1000 runs. Generally the experimental uncertainty calculated as a standard deviation from these 10 independent mean values, gives a sensible and interpretable indication of the relative positions of the log-2 binned distributions of times-to-completion.

In the case of the potted data shown for mean times to completion in Figure 4, 100 independent batches of 1000 independent runs were used. The mean and standard deviation are shown on the plots for given number of opinions Q and fixed system size.

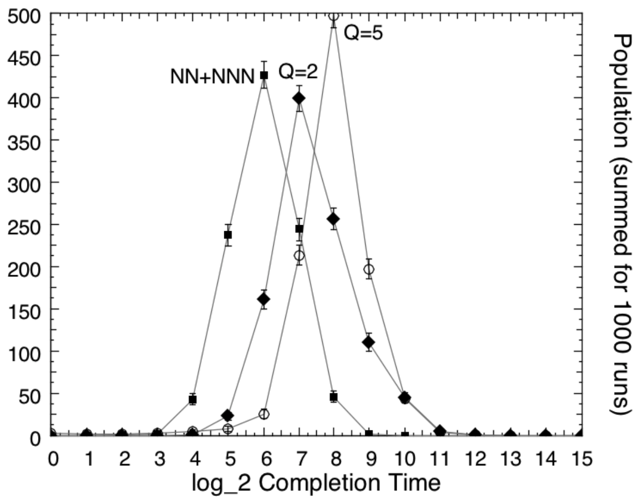

Figure 3: Distributions of completion times on log-2 histogrammed scale averaged over 10 batches of 1000 independent runs. Central $Q \equiv 2$ case (diamond plot symbols) on 64×64 sized mesh is reference case. Open symbols shows increasing Q to 5 and square symbols shows inclusion of next nearest neighbours.

Figure 3 shows the histogrammed distribution from 1000 independent model runs on a 64×64 model system. The distribution is essentially Poissonian, but the independent variable is shown on a log-base-2 scale. The central curve is for the usual two opinion ($Q \equiv 2$ case and the two distributions either side show a shift in the mean and median time to consensus completion. Increasing the number of initial opinions Q to 5 lengthens the mean time to achieving consensus. Allowing the next-nearest neighbouring voters as well as the nearest neighbours to be swayed by the agreeing pair of influencers causes the system to reach consensus more quickly and shifts the distribution to the left.

Figure 4 shows the variation of the average completion time to consensus for the model system as the number Q of different starting opinions in the mix is varied from the usual 2 up to 10.

5 Discussion of Observations

The effect of high Q is a monotonically rising time to consensus, although it is slower than linear. So adding

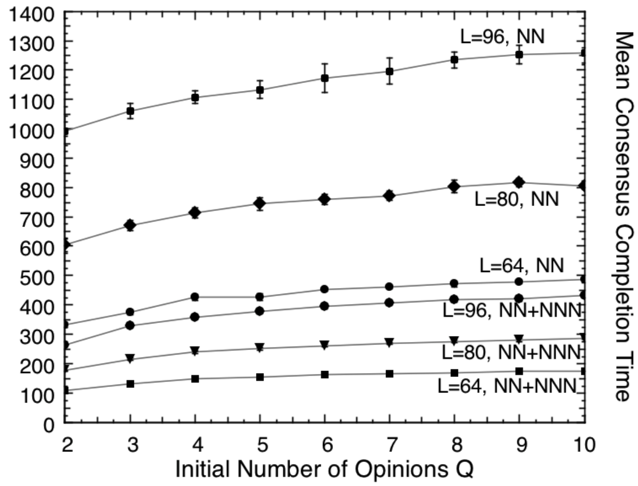

Figure 4: Plot of mean completion times for different numbers Q of initial opinions (or parties) in the mix.

extra political parties has a slowing effect but the slowing is diminishing. The addition of a third extra (opinion) party makes a lot more of a difference than adding a tenth (opinion) party does.

Note also that as expected the larger the system size, the longer it takes to achieve consensus. This is the same $O(N^2)$ effect that manifests itself in the computational time complexity of the simulation as discussed in Section 3 above.

Allowing next-nearest neighbours to be swayed by the opinion of the Sznajd-pairs speeds up opinion propagation across the model and allows the system to achieve consensus faster.

Some insights into why these statistical bulk behaviours are observed can be made by examining screen-dumps and animations of a system evolving with multiple initial opinions present.

Figure 5 shows a close-up screen visualisation of a system that has been initialised with $Q \equiv 3$ initial opinions or parties. Whereas geometrically, two majority parties would gradually form large clumps of similar opinions, that would join together and coalesce using surface tension effects, a third party slows this process up by impeding progress as shown where the blue minority opinion clumps are impeding the others.

Figure 6 shows how a model system with $Q \equiv 6$ original opinions progresses to two competing major groups as minorities are swallowed up by surface tension effects and suppression of viable fluctuations.

It is desirable to find a suitable metric that quantifies the level of agreement or consensus in the system. The number of voters with the same opinion is less use-

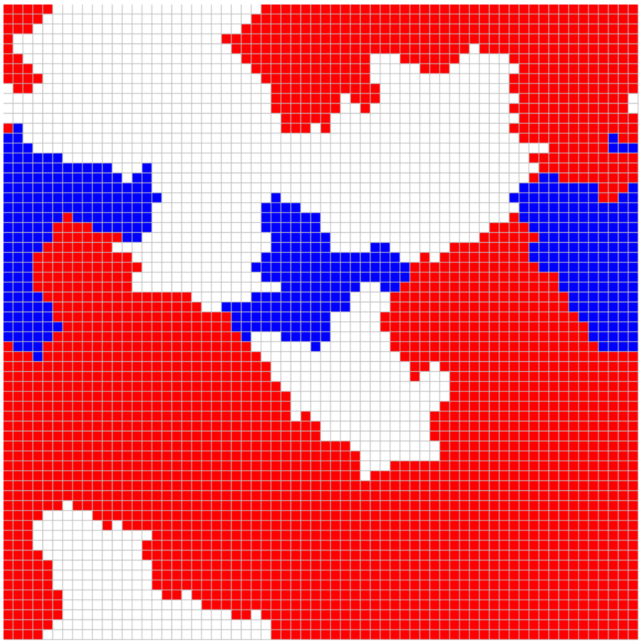

Figure 5: Screen-dump of a ($Q \equiv 3$) three-party(opinion) system showing how the blue minority is preventing clumps on majority opinion holders from joining up and achieving consensus.

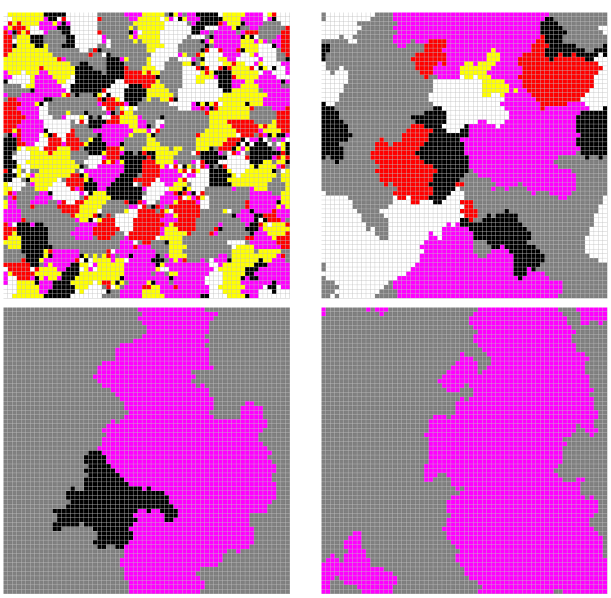

Figure 6: Sznajd Model on 64×64 periodic lattice with initially $Q \equiv 6$ opinions showing how minority opinions/parties are swallowed up by the majorities via surface tension effects and fluctuation suppression.

ful when there are more than $Q = 2$ possible opinions present in the system. A more useful metric would involve the number of separate clusters of isolated opinion holders. Counting the clusters in the system is computationally expensive however, so a cheaper and easier metric is related to the "energy of the system or the fraction of like-like bonds on the mesh. More simply put this is the fraction of possible neighbouring voter-voter relationships that are in agreement. For the square lattice, with four nearest neighbours for each voter, this gives two unique relationships per voter on average, and a value that varies from zero to $2 \times N = 2 \times L \times L$. We express this as a normalised fraction - the agreement fraction $f_a \in [0, 1]$

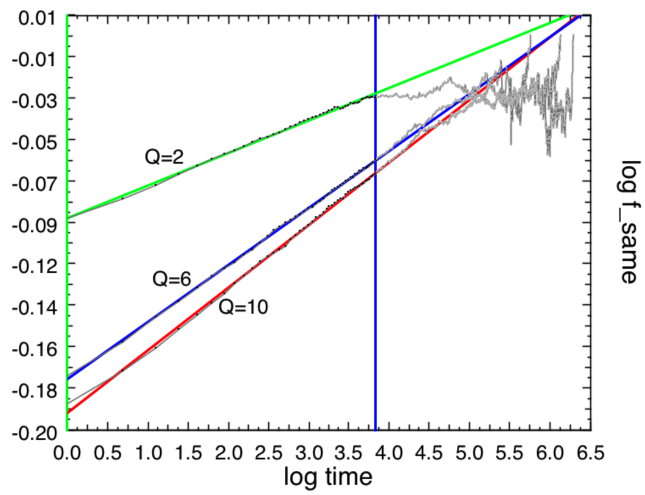

Figure 7: Fraction of voter relationships in agreement (like-like bonds) as a function of simulation time, shown on log-log scale for $Q = 2, 6, 10$ on a $N = 64 \times 64$ model system.

Figure 7 shows the time evolution of agreement fraction for the $N = 64 \times 64$ model system for $Q = 2, 6, 10$ on a log-log scale. The data have been averaged over 100 independent runs each of different starting conditions and converge remarkably to very straight lines at the early times. Uncertainties are similar to the indicative fluctuations shown in the later parts of the curve. This straight line fit (on the log-log scale) implies that $f_a \approx t^{\epsilon}$ where the exponent $\epsilon \equiv \epsilon(Q)$ and is an increasing function of Q.

Our data sets are not large enough to determine this functional dependence, but scanning over $Q = 2, ..., 10$ a very good fit is obtained from the relationship $\epsilon \approx (Q-2)^{\frac{1}{8}}$. The power fits to $Q-2$ rather than Q which we might guess is related to an excess of opinions over the normal 2 which is the minimum the models requires to work at all. The fitted exponent is 0.125 ± 0.01 which we postulate might in fact be $\frac{1}{8}$ arising from the geometry of the nearest and next-nearest neighbours on a square lattice.

This result is somewhat preliminary and requires further testing on larger model systems and with different geometries. However it does give a quantitative rationale for the consensus completion times increasing systematically with the number of different opinions or political parties in the original start configurations.

The curves shown in Figure 4 shows how consensus completion time t varies with number of opinions present Q for various system size and with influence neighbourhoods. This data yields straight lines on a log-log plot suggesting the relationship: $t = AQ^{\zeta}$. The effect of reducing the completion time when the influence neighbour hood is increased is confirmed. However it is also clear that there is a monotonically increasing relationship between the scale factors A in this equation so that $A = A(L)$. The data from the three system sizes $L = 64, 80, 96$ are consistent with an L^4 dependence which is also consistent with the data presented for the cpu time complexity in Figure 2. The exponent ζ in this equation is approximately 0.16 ± 0.01 although more data is required to confirm this is a rational fraction such as $\frac{1}{6}$.

6 Conclusions

We have described some detailed numerical simulations of the Sznajd model of opinion formation and have shown some definite effects when the number of initial opinions or political parties in the model system is increased. We have shown that as might be expected the time to completion increases when the system size is increased, but also that there is a definite relationship between the number of initial opinions or parties and the time to completion. Numerical simulations and empirical fitting have yielded some definite predictive formulae for these effects and suggest a number of other effects to be further investigated in opinion formation models such as this.

A most interesting conclusion is that while it might be expected that adding extra opinions or parties slows down the time to achieve consensus it does it in a very specific and non-linear manner. The additional "confusion" in a real voting or political system initially has a marked but diminishing effect on the time to completion. This measurements tell us that the third party makes a big difference but adding the tenth does not. This has some significance in considering the effect of independent political candidates standing against the two or three main parties commonly found in major democracies. It is also significant for studying the opinion dynamics in less mature democracies where there may be a larger number of political parties with no obvious initial majority.

We have started with unbiased populations where no one party or opinion has an advantage initially. It would be possible to model more specific political voting situations where the re is a definite initial leader. The model would also support the study of biased trajectories along the road to consensus by examining what happens if two large dominating parties are perturbed by a third minority. It could also be used to examine surprise swing effects when a shock bias is introduced - one party or opinion suddenly becomes unpopular at a global level due to media influence for example.

Finally, the model is attractive since it captures an essential complex behaviour exhibited in real voting systems without the need for a large number of arbitrary or adjustable parameters.

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