Assessing probabilistic forecasts of volcanic eruption onsets

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Probabilistic Forecasts

- Typically presented as:

“The probability of an eruption occurring in the next X years is Y”

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Bets Mt Taranaki blowing top in 50 years

By VIRGINIA WINDER

IF betting agencies took punts on natural disasters, you'd be wise to put your money on New Zealand's Mt Taranaki blowing its top in the next 50 years.

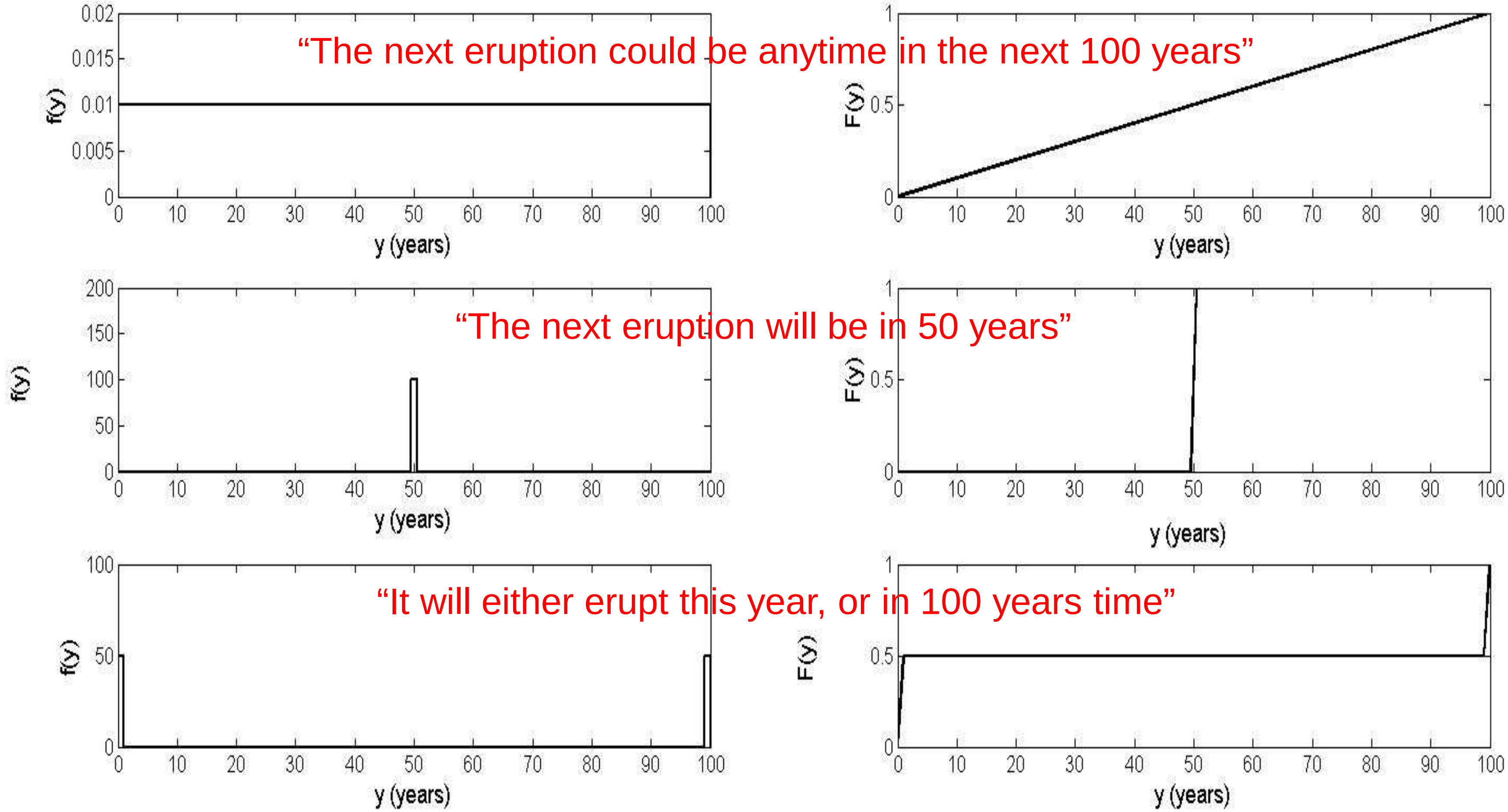
You might not be around to collect the winnings, but your descendants would be toasting your foresight with an expensive bottle of champers, probably far from the poison ash fallout. Perhaps Paris.

Massey University scientists have fresh evidence the Egmont Volcano, as it's known in geological terms, may have a history of violence even worse than previously thought.

So bad, that if it continues its destructive track record, there's 50-50 chance it will act up before 2057.

The problem with X and Y

- All of these have a 50-50 probability of an eruption within 50 years:

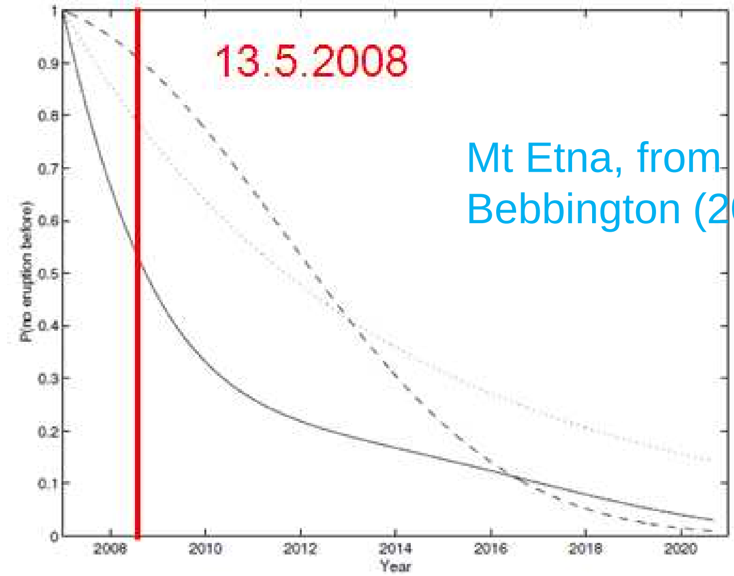


While these forecasts have the same mean, the variance differs considerably!

Forecast Distribution

Instead, express forecast as
$F(x) = \text{Prob}(X \leq x)$, and suppose
$X = x_1$ is observed.

Then the quantile $F(x_1) \sim U(0,1)$
Random variable if the forecast
F correctly describes the process.



Mt Etna, from Bebbington (2007)

Figure 8. Mount Etna: forecast probability distribution of the next onset. Solid line = time-predictable model, dashed line = size-predictable model and dotted line = interonset model.

If we have multiple observations
$x_1, x_2, \ldots x_n$, we can test for departure from U(0,1):

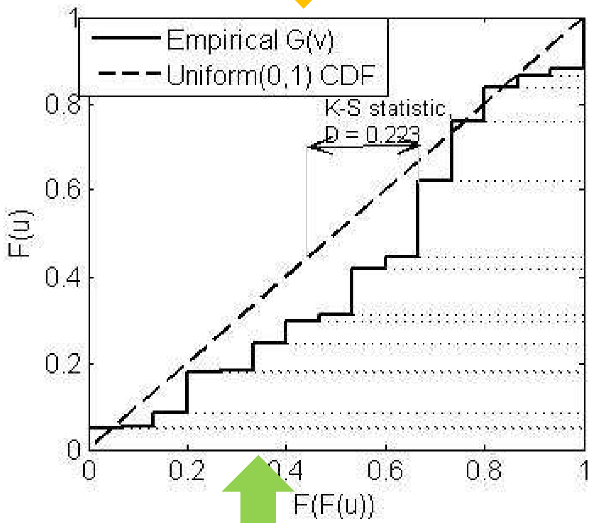
Use, e.g., Kolmogorov-Smirnov test.

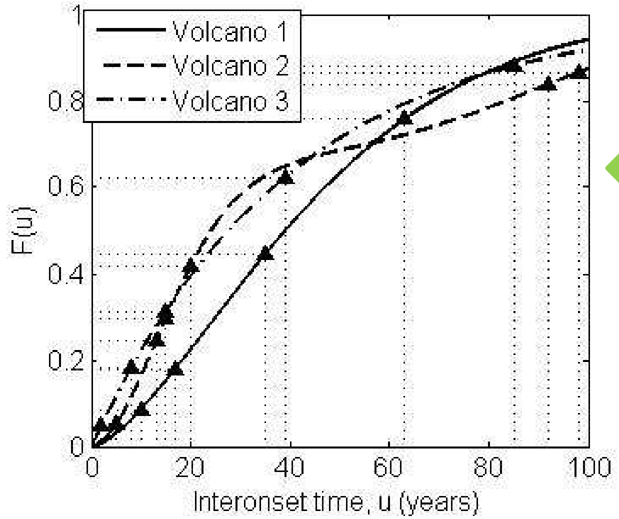
Include open (incomplete repose) forecasts using Kaplan-Meier product limit estimate.

Observations need not be from same volcano, or same method, allowing aggregation of forecasts for significance testing.

Schematically ...

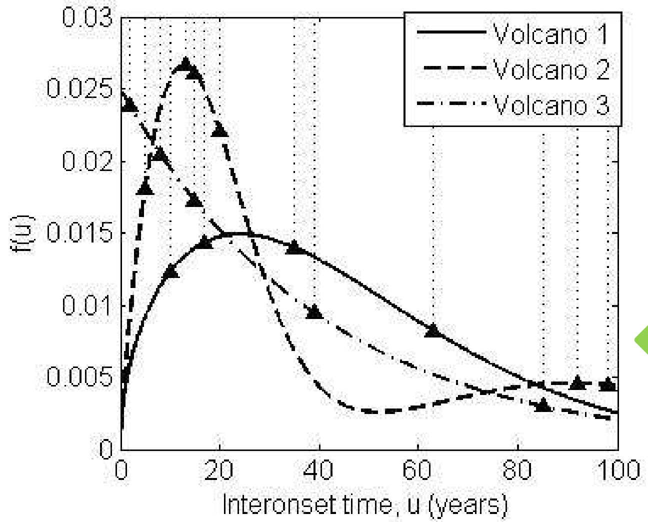
Axes reversed from usual practice to aid comprehension.





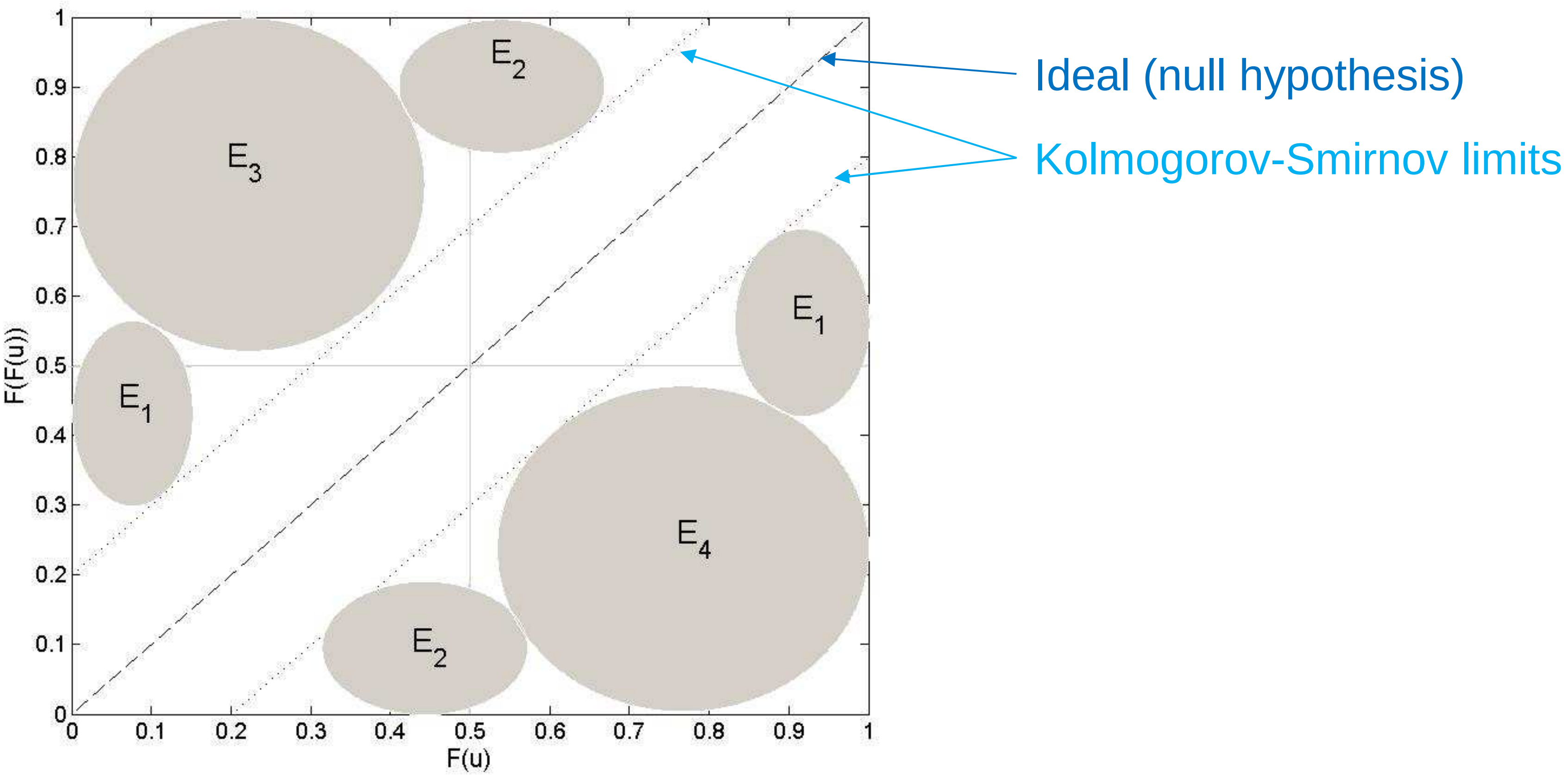
2. The quantile F(u) at each observation becomes an observed value on the probability forecast scale.

3. These values are reasonably close to a uniform (0,1) distribution



1. Forecast densities f(u) for 3 volcanoes, and observed reposes (triangles)

Diagnostics



<u>Types of Error:</u>

E_1 observations more variable than forecasts
E_2 observations less variable than forecasts
E_3 observations on average shorter than forecasts
E_4 observations on average longer than forecasts.

A Gallery of Flakes, Ruffians and Oddballs

Model	Volcano	Data (source)	Subsequent eruptions (Simkin & Seibert 2002-)	Reference
Weibull renewal	Aso	1800-1980 (Simkin *et al.* 1981)	1981, 1983, 1984, 1988, 1989, 1992, 1994, 2003, 2004, 2005, 2008, 2011	Bebbington & Lai (1996a)
	Etna	1605-1978 (Simkin *et al.* 1981)	1980, 1981, 1981, 1981, 1983, 1984, 1984, 1985, 1985, 1986, 1986, 1987, 1988, 1989, 1989, 1991, 1993, 1993, 1994, 2001, 2001, 2002, 2003, 2004, 2004, 2005, 2006, 2007, [illegible]	Bebbington & Lai (1996a)
	Kilauea	1800-1979 (Simkin *et al.* 1981)	1980, 1982, 1982, 1983	Bebbington & Lai (1996a)
	St Helens	1831-1980 (Simkin *et al.* 1981)	1989, 1990, 2004	Bebbington & Lai (1996a)
	Yake-Dake	1910-1962 (Simkin *et al.* 1981)	1995	Bebbington & Lai (1996a)
	Ruapehu	1861-1984 (Latter 1985)	1995, 1996, 1997, 2006, 2007, 2007	Bebbington & Lai (1996b)
	Ngauruhoe	1839-1984 (Latter 1985)		Bebbington & Lai (1996b)
Generalized negative binomial	Sangay (Ecuador)	1728-1976 (Simkin *et al.* 1981)	1983, 1988, 1995, 2004, 2004, 2004, 2005, 2006, 2008, 2008, 2009, 2009	Bebbington & Lai (1998)
Hierarchical	Taveuni (Fiji)	10ka BP-340 BP		Cronin *et al.* (2001)
HMM	Etna (flank)	1600-2006 (various)	2008, 2010	Bebbington (2007)
Weibull mixture renewal	Taranaki	10ka BP - 1860		Turner *et al.* (2008,2009)
Volume history	Etna (flank)	1600-2006 (various)	2008, 2010	Bebbington (2008)
	Mauna Loa	? - 1984 (USGS)		Bebbington (2008)
Trend renewal process (VEI 2+)	Aso	1230-2009 (Siebert & Simkin 2002-)	2011	Bebbington (2010)
	Etna (flank)	1603-2009 (Siebert & Simkin 2002-)	2010	Bebbington (2010)
	Fournaise	1640-2009 (Siebert & Simkin 2002-)	2010	Bebbington (2010)
	Gamalama	1643-2009 (Siebert & Simkin 2002-)		Bebbington (2010)
	Kliuchevskoi	1697-2009 (Siebert & Simkin 2002-)	2011	Bebbington (2010)
	Lamongan	1799-2009 (Siebert & Simkin 2002-)		Bebbington (2010)
	Marapi	1770-2009 (Siebert & Simkin 2002-)		Bebbington (2010)
	Merapi	1548-2009 (Siebert & Simkin 2002-)	2010	Bebbington (2010)
	Ngauruhoe	1841-2009 (Siebert & Simkin 2002-)		Bebbington (2010)
	Ruapehu	1861-2009 (Siebert & Simkin 2002-)		Bebbington (2010)
	Semeru	1818-2009 (Siebert & Simkin 2002-)		Bebbington (2010)
	Tengger	1804-2009 (Siebert & Simkin 2002-)	2010	Bebbington (2010)
Self-exciting	AVF	250ka BP-AD1250		Bebbington & Cronin (2011)

No access to 2nd Ed. (Pre-WWW)

Includes `summit' eruptions

Original m/s submitted 1994, revision 1995

1983 eruption "continuing"

Data reclassification from 1st Ed.

1967 eruption "continuing"

Some weaponry

- Weibull renewal process $F(u \mid H_T) = 1 - \exp\left[-(\beta u)^{\alpha}\right]$
  - allows for clustering or regularity relative to Poisson process
- Trend renewal process $F(u \mid H_T) = F_0\left[\Psi(s+u) - \Psi(s)\right]$
  - $\Psi()$ is a time-rescaling, allowing for the incorporation of clustering, trends, wax-and-wane, cycles, ...

s = time of last eruption

- Volume-history model

$$F(u \mid H_T) = \exp\left(-\int_s^{s+u} \exp\left\{\alpha + \nu\left[\rho t - U(t)\right]\right\} dt\right)$$

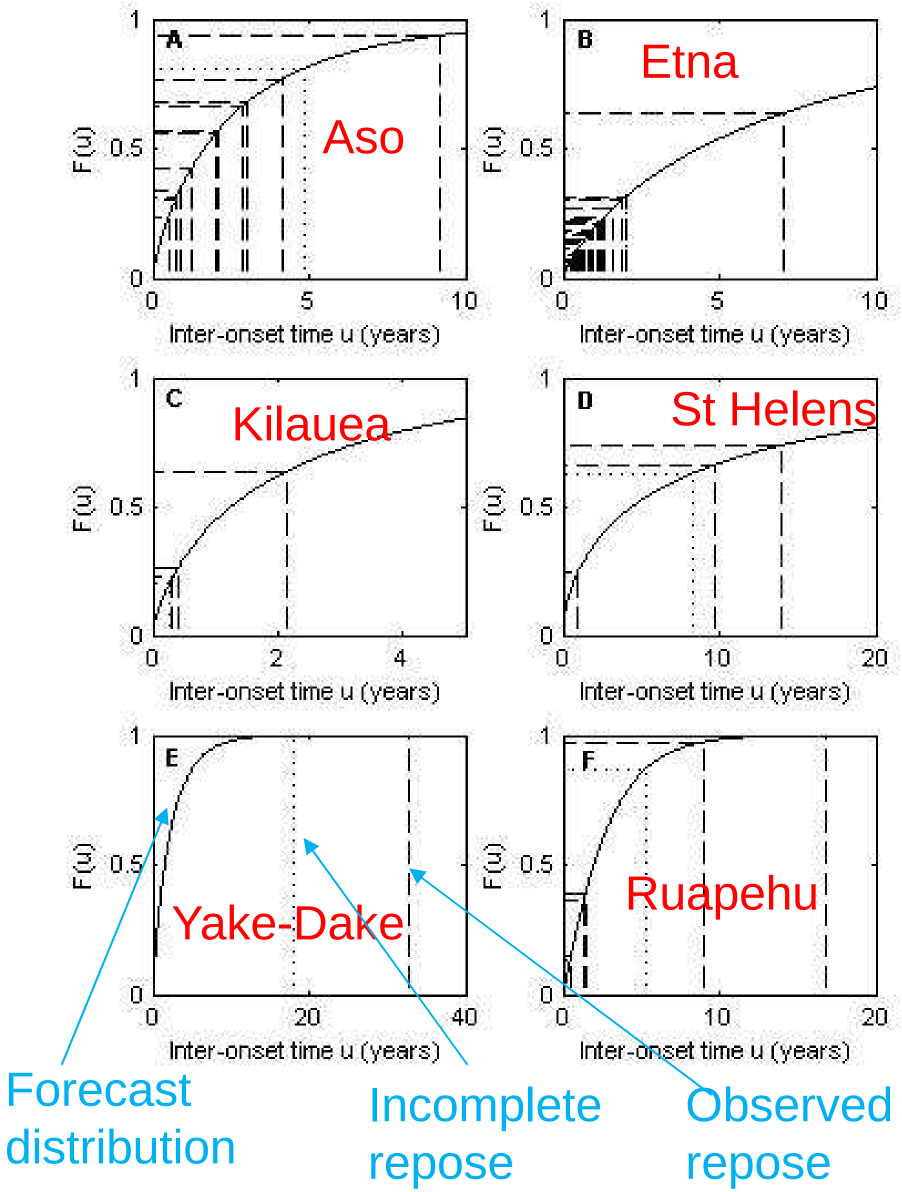
$U(t)$ = total eruptive volume at time t

  - (generalized time-predictable model)
- Generalized negative binomial

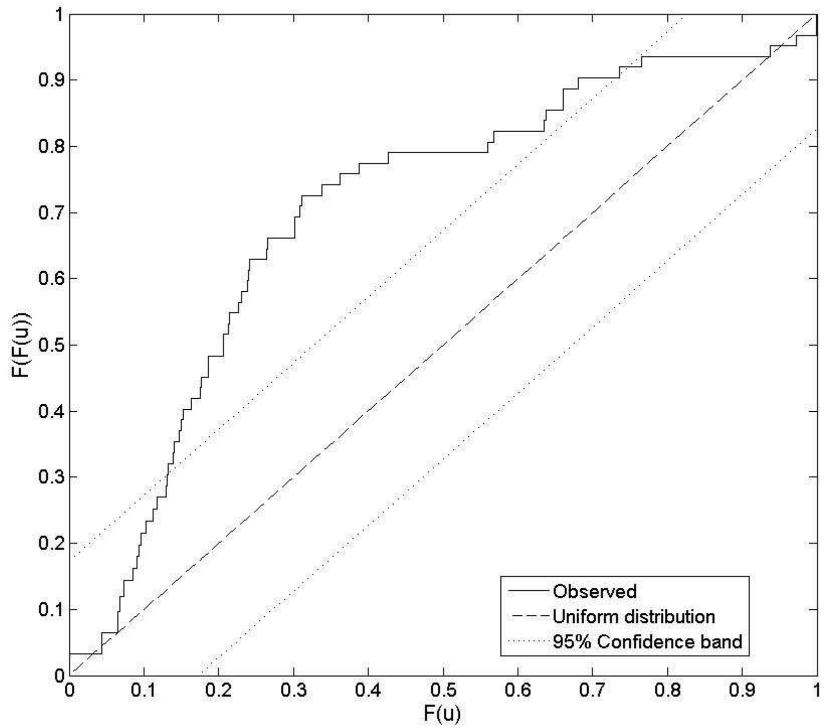
$$F\left(\llbracket u \rrbracket \mid H_T\right) = 1 - \lambda\kappa\left[1 - (1-\lambda)\kappa\right]^{\llbracket u \rrbracket - 1}$$

  - introduces correlation $1 - \kappa$ between events in discrete time (years)

Results: Weibull Renewal Process



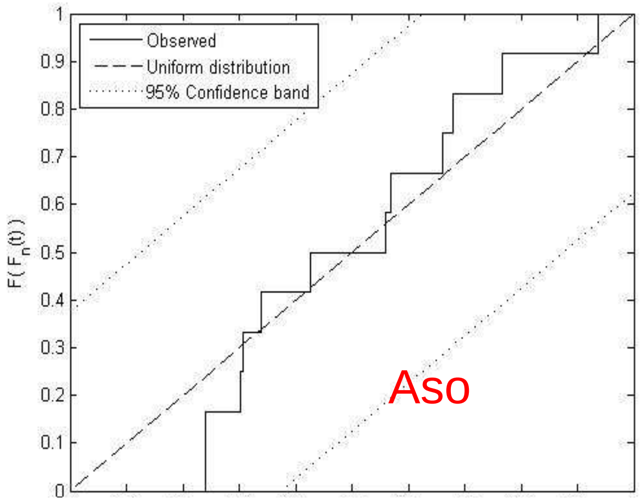
And the test results, please ...

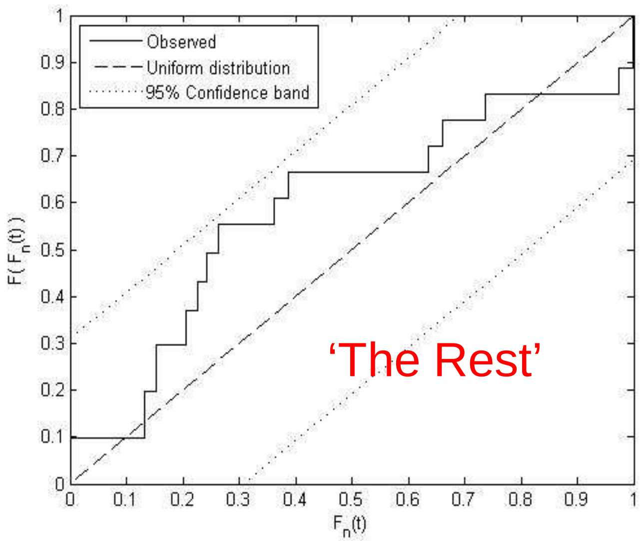


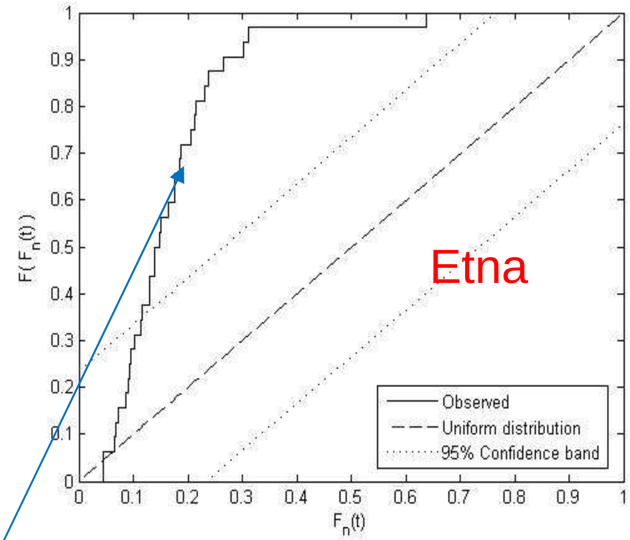
Hmmm....

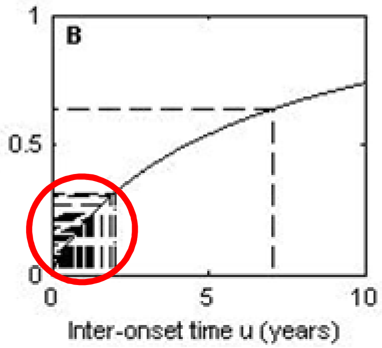
Results: Weibull Renewal Process (2)

A little disaggregation of the results ... and voila!



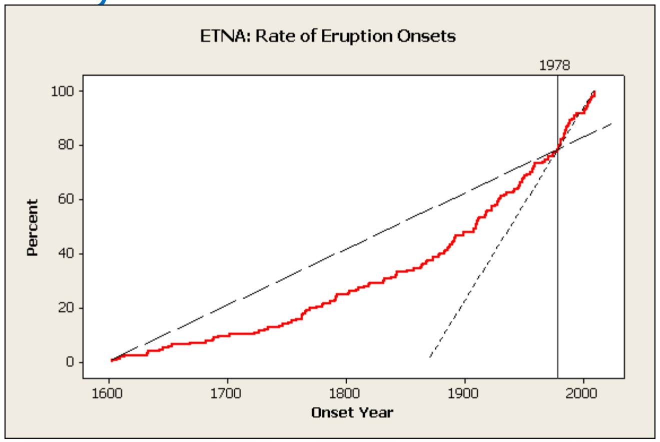






Renewal models assume that the average rate of onsets is constant through time

- Increasing observation prob. of summit eruptions?
- Recent activity above historical norms?



Trend Renewal Process (Twelve angry volcanoes)

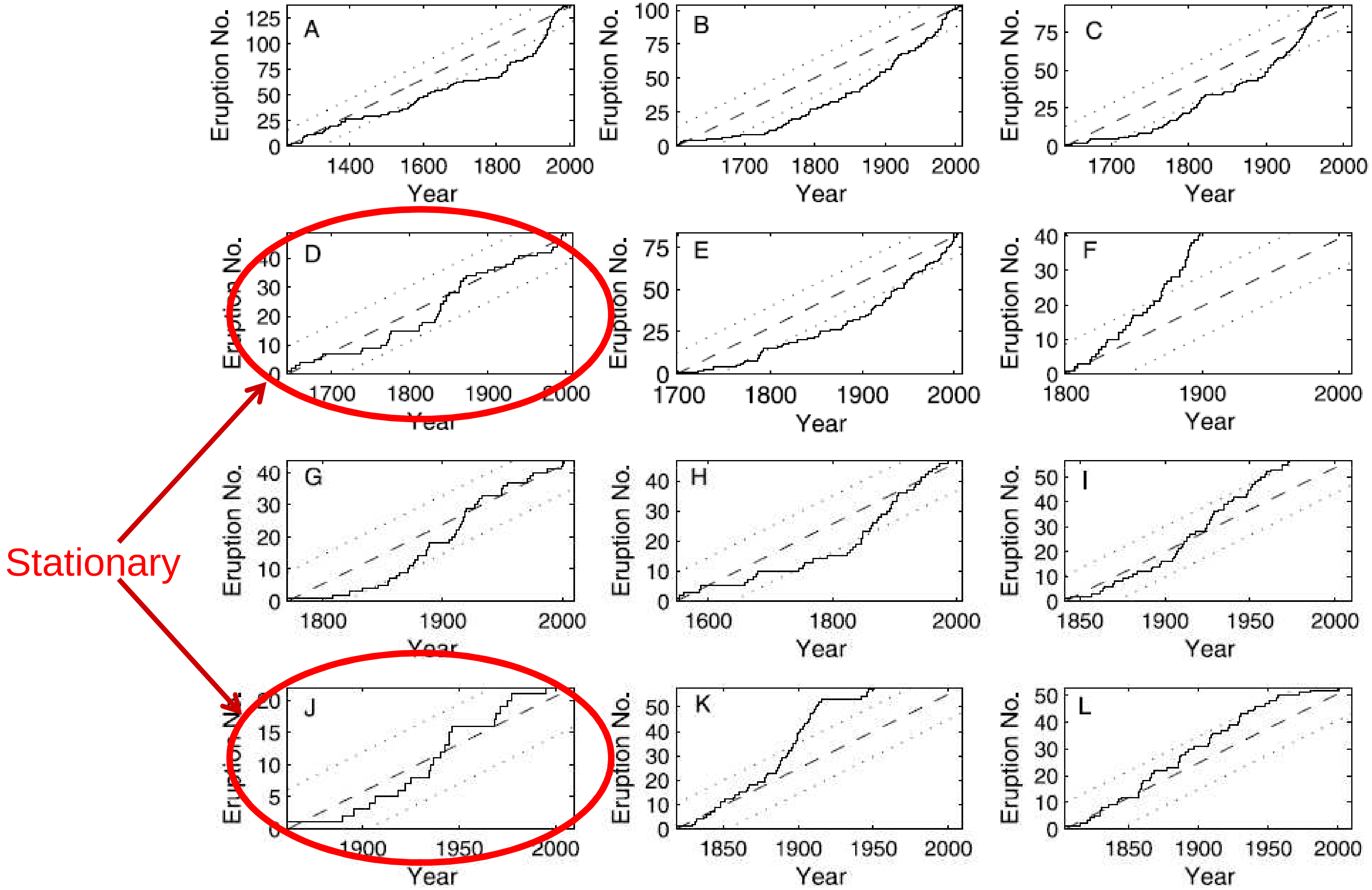


Figure 2. Cumulative number of eruptions (solid line), average rate (dashed line) and 95% confidence band for a stationary rate process (dotted lines). (a) Aso, (b) Etna, (c) Fournaise, (d) Gamalama, (e) Kliuchevskoi, (f) Lamongan, (g) Marapi, (h) Merapi, (i) Ngauruhoe, (j) Ruapehu, (k) Semeru, (l) Tengger.

Trend renewal process: Best models

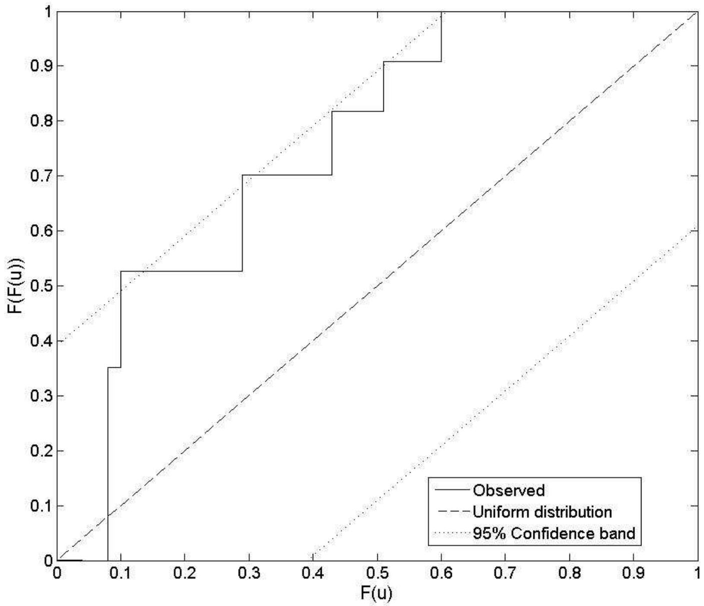
Table 7. Best AIC Models and Forecasts[a]

Volcano	Model	Number of Parameters	AIC	Quantiles (u) of $P(t_{n+1} > u) \mid P(t_{n+1} > 2009.2)$ 95th	UQ	Median	LQ	5th
Aso	Cyclic, $\eta = 1$	6	−710.8	2009.4	2010.1	2011.3 **2011.4**	2013.3	2018.0
Etna	Weibull, $\eta = 1$	3	−459.6	2009.3	2009.8 **2010.3**	2010.5	2011.7	2014.6
Fournaise	Cylic, $\eta = 1$, $\mu = 0$	5	−414.5	2009.5	2010.5 **2010.8**	2012.3	2015.2	2021.8
Gamalama	Cyclic, $\eta = 1$, $\phi = 0$	5	−290.0	2009.5	2010.8 **?**	2012.9	2016.4	2024.2
Kliuchevskoi	Weibull, $\mu = 0$	3	−361.3	2009.4	2010.0	2010.8 **2011.3**	2012.0	2014.6
Lamongan	Gamma, $\eta = 1$, $\mu = 0$	3	−175.7	2010.2 **?**	2015.1	2024.6	2042.8	2082.5
Marapi	Gamma, $\eta = 1$, $\mu = 0$	3	−219.5	2009.7 **?**	2012.0	2016.0	2023.7	2046.8
Merapi	Gamma, $\eta = 1$	4	−293.9	2010.1 **2010.8**	2014.1	2021.1	2033.7	2064.1
Ngauruhoe	Gamma, $\eta = 1$	4	−223.6	2009.6	2011.2 **?**	2014.0	2018.8	2030.1
Ruapehu	Gamma, $\eta = 1$, $\mu = 0$	3	−119.2	2010.0 **?**	2013.9	2020.7	2033.4	2065.1
Semeru	Gamma, $\eta = 1$, $\mu = 0$	3	−216.3	2010.1	2014.4	2022.9	2039.7	2075.1
Tengger	Gamma, $\eta = 1$, $\mu = 0$	3	−238.3	2010.0 **2010.9**	2013.5	2020.2	2033.8	2093.2

[a]LQ and UQ are lower and upper quantiles, respectively.

The 1967 eruption of Semeru is continuing

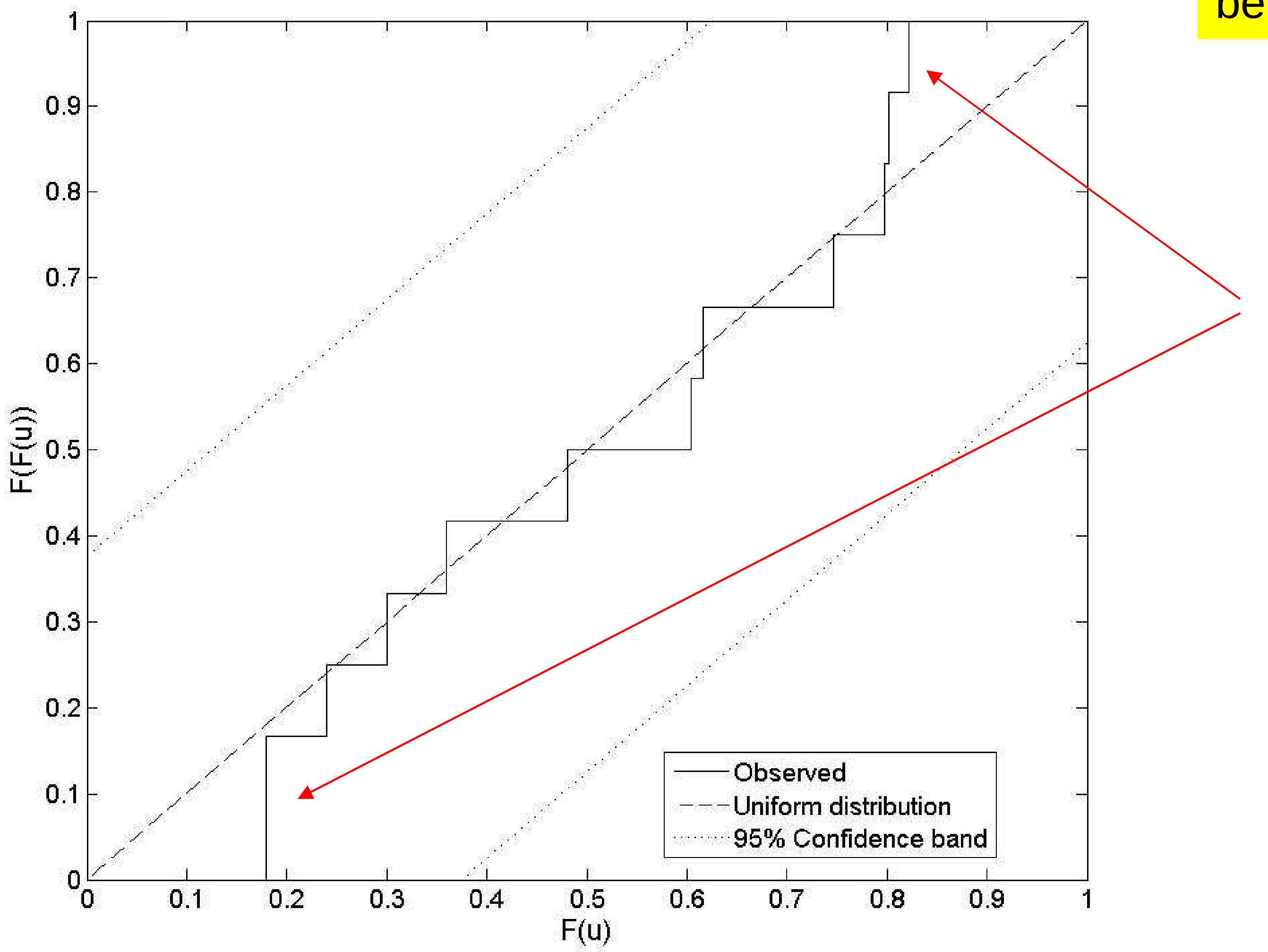
Model potentially incorporates cycles, trends or wax/wane of activity through time-rescaling



Generalized Negative Binomial (Mt Sangay)

NB: This is a discrete time (years) model

Focus is clustering of eruptions (correlation between reposes)



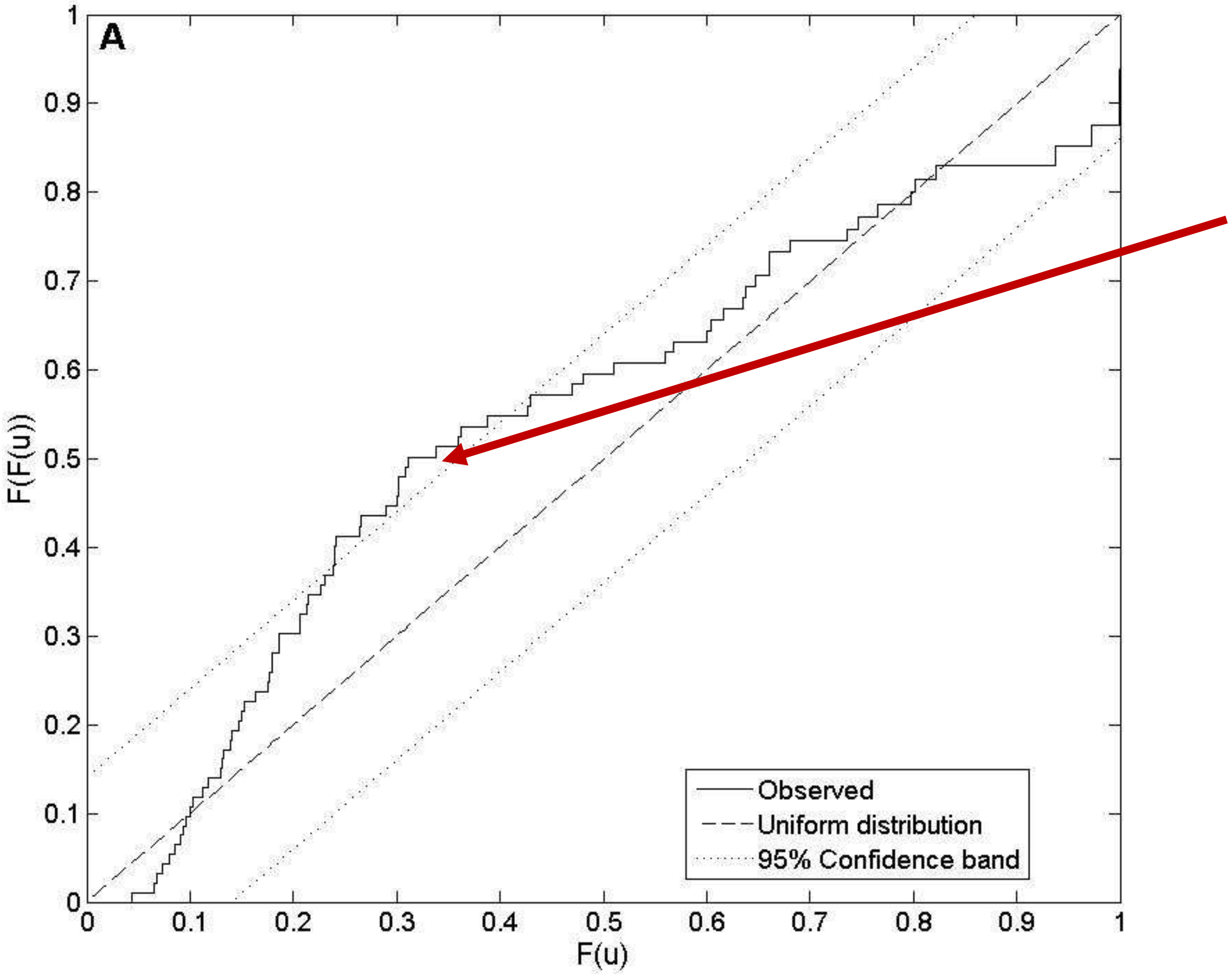
Possible tendency to over-estimate variability?

Aggregating

- A simple aggregation of the empirical repose distributions can be used to assess the quality of forecasts for:
  - a particular volcano, and/or
  - a particular model, and/or
  - a particular forecaster ☺

NOTE: the result is implicitly weighted by the number of post-forecast eruptions for each forecast.

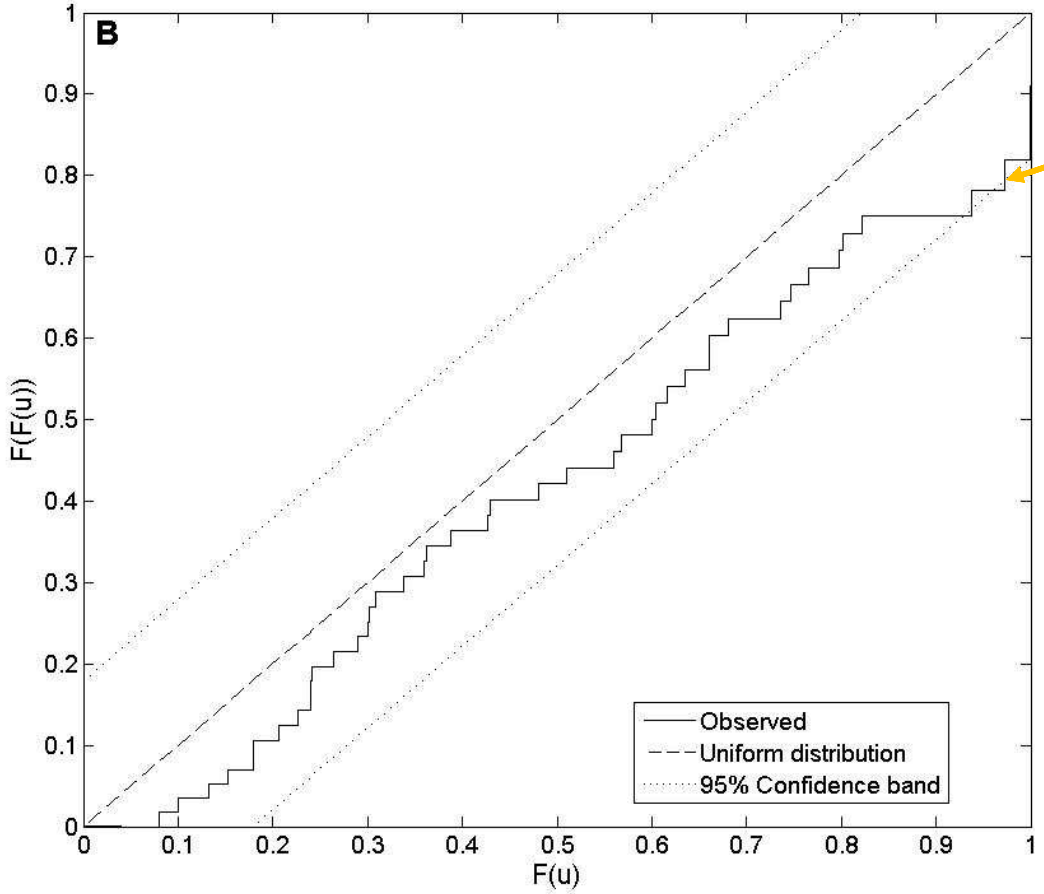
How good is this guy?



Not great.

But ...

How good is this guy on non-Sicilian volcanoes?

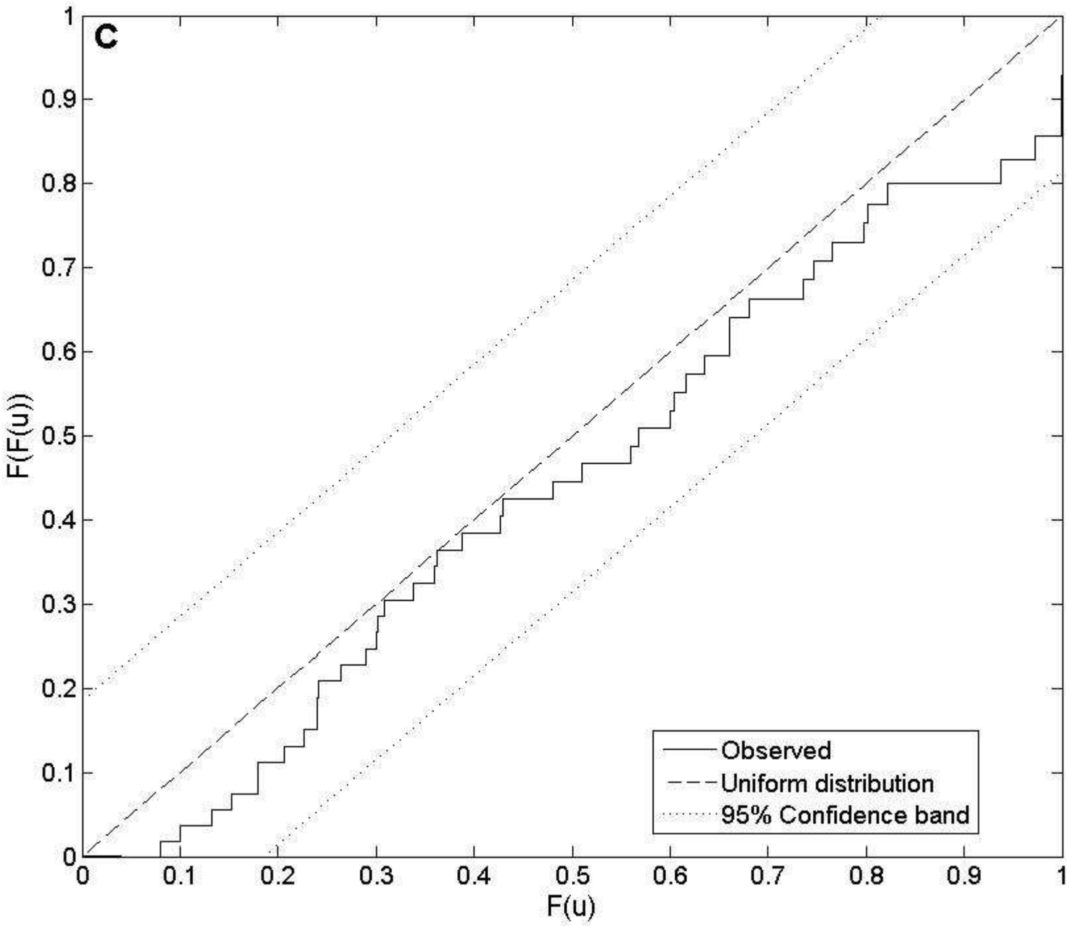


Comme ce, comme sa

- Open long reposes?

...if we omit Etna.

And ...



...any other volcano that hasn't erupted since 1850AD

- The method:-
  - provides a tool for validating models, and in particular the inherent degree of uncertainty in the forecasts
    - from a suite of eruption onset forecasts1995-2011, the eruption onsets of Mt Etna were identified as being inconsistent with the forecasts, due to a change in eruptive behaviour
  - assesses whether forecasts are correct ***in the context of the included uncertainty***
    - It does NOT rank different forecasts
  - provides a possible means of weighting models in an ensemble forecast.

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