

Credit Crunch as an Optimal Decision of Risk-Averse  
Banks under Uncertainty.

DRAFT FOR COMMENTS

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”Not surprisingly, in response to past laxity, a weakening economy and general economic uncertainty, banks have tightened their lending terms and conditions.”

Alan Greenspan, March 7, 2001

## 1 Introduction and Motivation

The literature on credit cycles emphasizes the role of asymmetric information or contract enforcement in modeling the credit market behavior. Credit constraints arise naturally because banks want a collateral for any size of a loan. The value of collateral therefore plays a crucial role in determining the supply of credit. For a given expected return, net worth fluctuations imply changes in credit constraints with an opposite phase and changes in amplitude and persistence of the effects of monetary shocks with the same phase. Yet the rise in persistence and amplitude is not a result of the propagation through an objective functions in any of the models. It occurs because the constraints preventing firm’s borrowing on viable projects are tightened when the net worth values drop. The *amplitude* rises because in ”good times” firms are more productive and at the same time less constrained. The response to shocks is *persistent* because firms are constrained for many periods. Persistence increases (decreases) when firms are more (less) constrained, and its behavior is asymmetric when firm’s production function is concave<sup>1</sup>. In other words, the current literature introduces dynamics into the behavior of credit by imposing increasingly more sophisticated constraints on the behavior of agents in the economy.

Propagation through constraints implies that a removal of the constraint leads to a more efficient result. It is sometimes hard to justify because the costs of removing the constraint often appear to be smaller than the benefits of moving to a more efficient outcome. In terms of the credit crunch literature, it may appear that the costs of mitigating the informational asymmetries which motivate the existence of net worth constraints (and so lead to a credit crunch) are smaller than the aggregate effects of inefficiently low credit. If this was correct we should observe actions aimed at mitigating these inefficiencies, either by the governments or by firms that suffer most. But we do not observe this. One could argue that the constraints implied by the asymmetric information paradigm do not account for a large part of the credit cycle and that an alternative explanation is needed. The lack of actions to ease the problems of asymmetric information is observationally equivalent to a hypothesis that the credit cycle is an optimal phenomenon.

In this paper we offer an explanation for the cyclical behavior of credit without assuming the presence of contract enforcement or asymmetric information in its commonly used meaning. In effect, we show that even after eliminating all asymmetric

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<sup>1</sup>A very constrained firm reacts more on a margin than a slightly constrained firm if MPK is declining.

information and contract enforcement problems, one would still observe credit cycles. It is not the poor information among the agents (firm vs. bank) and hence the net worth fluctuation that leads to credit cycles, but the unavailability of information about the future returns which causes it.

In the model, a risk-averse bank maximizes an expected discounted sum of its "utility function" (a concave transformation) of profits, which acts as a proxy for various commonly used assumptions in the literature that aim to introduce non-linearities into bank's profit function. When solved exactly (rather than with a linearization), the variance of expected returns affects the level of credit in two ways. First, the steady state level of credit is a decreasing function of the uncertainty. Second, transition to a new steady state is slower following an increase in the uncertainty. We show that when there are shocks to the variance of the returns on capital, banks will optimally contract or expand their loan portfolio even when the expected return does not change. Intuitively, it is the asymmetric information imbedded in the time dimension and the nature of decisions banks make that can account for a cyclical behavior of credit.

The results of this paper suggest the existence of another channel through which aggregate shocks affect firm behavior and exacerbate the credit cycles. This channel is equally important for small and large firms. It therefore has a potential to explain the reasons behind the problems with identifying asymmetric effects of shocks on small and large firms predicted by the asymmetric information models. One obvious criticism of the model presented here is that it disregards the existence of other capital markets which might be accessible by large firms. However, as long as the lenders in those markets include risk in pricing their credit, their behavior will be similar to that of banks in my model and the impulse response functions described below should be qualitatively the same. Also, it nests the case when bank has an option of two assets as long as the second type of asset is stationary (see section 3.1).

The paper is structured as follows. Section 2 discusses the current literature on credit crunches/cycles. Section 3 presents and section 4 solves the deterministic and stochastic versions of the model. Section 5 describes the calibration and section 6 the simulations. Section 7 presents the implications for the monetary policy. Section 8 concludes.

## 2 A highly selective literature review

The asymmetric information paradigm is based on an assumption of costly state verification from the side of the bank. The monitoring costs banks incur in order to learn about the state (e.g. a value of an idiosyncratic shock the firm receives) represents an inefficiency. This cuts off some borrowers that would, in a first-best scenario, make viable investments.

Townsend (1979) derives optimal contracts in the presence of a costly state verification problem. Townsend (1982) shows that in environments with private information, multiperiod contracts between agents with identical discount rates can bring the private

information solution arbitrarily close to the perfect information solution as the time horizon of the contract increases. This has two implications for the models of banking sector. It offers a way to rationalize long-term relationship banking and it also suggests that the private information barrier is alleviated under optimal multiperiod contracting. Thomas and Worall (1990) show that the agency problem disappears when the financial relationship is enduring and the discount factor is close to one.

The net worth was first used as a vehicle for introducing this type of asymmetric information friction into credit/capital behavior by Bernanke and Gertler (1989). Kiyotaki and Moore (1995) show how credit constraints interact with aggregate economic activity. Borrowers' credit limits can be affected by the prices of the collateralized assets but, at the same time, these prices are affected by the credit limits, producing credit cycles. Carlstrom and Fuerst (1997) bring the costly state verification problem into a general equilibrium setting. Bernanke, Gertler and Gilchrist (1998) introduce the financial accelerator into a quantitative business cycle framework.

Gertler and Gilchrist (1992) and Bernanke, Gertler and Gilchrist (1999) provide interesting reviews of the financial accelerator literature in which the assumption of the asymmetric information is introduced through assuming that firms have to possess certain amount of net worth in order to be eligible for a loan. Warner and Georges (2001) use stock market returns data to examine whether small firms benefit more significantly following a Fed shock monetary easing. They find no support for the hypothesis that small and large firms are affected differently by the monetary policy shocks. This contradicts the conclusions of an empirical paper by Gertler and Gilchrist (1994).

Less work was done in the credit crunch area, partly because the issue is not clearly defined. A large amount of work is empirical (focused on the period of 1988-1992), which in itself can not characterize credit cycles sufficiently to distinguish between credit supply and credit demand – a distinction crucial for specifying a credit crunch. The Council of Economic Advisors defines a credit crunch as a situation when the supply of credit is restricted below the range usually identified with prevailing market interest rates and the profitability of investment projects. Because the credit crunch is a supply phenomenon, it is difficult to capture with most of the current models of credit cycles in which the bank's decision problem simply does not allow for cycles in supply of credit (see section 2 below). Models which attribute credit cycles to various mechanisms will differ in their explanation of the causes of a credit crunch. It therefore comes as no surprise that credit crunch periods are hard to identify and some studies find that certain periods informally considered to be cases of severe credit tightness can actually be the result of a cyclical decline in credit demand (see, e.g. Pazarbasioglu (1997)).

On the empirical side, Ceyla Pazarbasioglu (1997) disentangles credit supply and demand during the Finnish banking crisis in 1991-1992 and finds that the observed decline in the volume of credit was primarily caused by the decline in credit demand, rejecting the credit crunch hypothesis. Hancock and Wilcox (1998) use 1989-1992 U.S.

state bank data to find that small banks react more strongly to their capital decline than the large banks, and that this decline has a bigger real impact due to their loan portfolio. Wagster (1999) uses Canadian, UK and US data on banks' balance sheets to test various hypotheses for the portfolio shift from loans to securities in 1989-1992.

On the theoretical side, Diaz-Gimenez, Prescott, Fitzgerald and Alvarez (1992) develop a computable general equilibrium model with an explicit role for the banking sector which, however, does not play a significant role. Berglof and Roland (1997) show that the credit crunches can coexist with soft budget constraints during the times of financial transition. Spiegel (1996) uses a monopolistically competitive model of foreign lending where fixed-premium deposit insurance movements in bad times create credit crunches. Little work has been done in the credit cycle literature using a heterogeneous agent assumption. In Carlström and Fuerst (1998), agents are only heterogeneous *ex-ante*. As Fachat (2000) shows, their model can not replicate as many stylized facts as a model without heterogeneous agents. Berka and Zimmermann (2001) show that in a computable dynamic general equilibrium model where agents are heterogeneous *ex-post*, credit crunches can occur and may be softened by monetary policy actions.

The second large body of asymmetric information literature works through the contract enforcement paradigm. We do not elaborate on it in this paper.

### 3 Model

The model I present here has several general characteristics. First, it is a stochastic dynamic partial equilibrium model with a representative agent. Lack of heterogeneity of the banks as well as borrowers is a simplifying assumption that makes the analysis more tractable and, more importantly, highlights the contribution of aggregate variables to credit behavior. Second, the stochastic environment plays an important role as a source of uncertainty, which affects the *levels* of variables<sup>2</sup>. Third, there is no asymmetric information between the agents in the economy.

#### 3.1 Supply of credit

The innovation of this paper is in modeling of the credit supply decision of the bank. The bank maximizes an expected discounted sum of concave transformations of future profits. The concave transformation implies that the bank takes into account the riskiness of its assets and liabilities, either voluntarily or by requirement. This can be modeled as accounting for (a) an incentive structure imposed on the management by risk-averse shareholders (see, e.g., Spiegel (1996)), (b) operating costs of managing the risk of its asset portfolio (some form of non-linearity in the cost of funds), (c) deposit insurance (Spiegel (1996)), (d) required reserve holdings against liquid liabilities,

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<sup>2</sup>With an approximate solution, variances of exogenous variables only affect variances of endogenous variables.

(e) Basle Accord requirements based on risk-adjusted capital adequacy ratios<sup>3</sup>, or (f) Value-at-risk constraints imposed voluntarily by the bank management. Empirically, risk aversion is one way of explaining the procyclical behavior of credit conditions (as opposed to terms) for a given interest rate.

All of the above modeling approaches make the profit function of the bank non-linear in one or more of its choice variables. Consequently, they induce risk-averse behavior. I do not specifically model the source of this risk-aversion in order to make the results more general and also because applying a transformation to the entire profit function makes it more analytically operational than adding a non-linearity to a specific component of it (see Appendix B for a discussion of this point). Bank's maximization problem is as follows:

$$\max_{\{A_t, D_t, C_t\}_0^\infty} E_0 \sum_{t=1}^{\infty} \beta^t \phi(\Pi_t) \quad (1)$$

subject to the following constraints:

$$\Pi_t = (1 + r_t)A_{t-1} - A_t - r_t^d D_t \quad (2)$$

$$D_t + C_t = A_t \quad (3)$$

$$C_t \geq \rho A_t \quad (4)$$

for given  $\{r_t\}, \{r_t^d\}, A_{-1}$ .

Let  $\Pi_t$  represent the profits and  $\phi(\cdot)$  the concave transformation ( $\phi'(\cdot) > 0, \phi''(\cdot) < 0$ ) discussed above. Let  $A_t, D_t, C_t$  represent assets, deposits and capital, respectively. The interest rates on loans and deposits are given by  $r_t$  and  $r_t^d$ , respectively. Let  $\rho$  represent a capital adequacy requirement. The profit function (2) with loans of one year maturity. Banks have to decide on the amount of loans one period before they receive a return on them (this is one way of introducing an intertemporal connection to an otherwise static problem). Banks pay an interest rate  $r^d$  on their deposits. Equation (3) equates assets and liabilities at all times, and the inequality (4) is a capital adequacy requirement imposed by the regulators<sup>4</sup>.

In this simplest version of the model, I assume the existence of only one type of asset and disregard the changes in the composition of bank's assets. If I interpret  $A_t$  purely as loans, this model nests a problem with two types of assets, so long as the other asset type is relatively stationary. Bernanke and Blinder (1992) provide evidence that following a negative monetary policy shock, banks shed securities upon impact while keeping loans unchanged. Loan volumes are adjusted only after 4 months. Securities holdings rebound about 6 months following a shock while loan volumes keep declining

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<sup>3</sup>Because these impose restrictions on market values of assets and capital, they impose aversion of the bank towards riskier types of loans as well as towards all categories of assets during periods of economic downturn - CHECK THIS IN THE ACCORD.

<sup>4</sup>I do not model the capital adequacy ratio  $\rho$  as dependent on risk. Instead, I assume the bank accounts for the risk with its "utility function" in profits. The role of the inequality constraint (4) is insignificant for an interior solution, which is the main focus of this paper.

throughout the time span of their analysis (24 months). While the model presented here does not allow for this richness in behavior, one can argue that, if the security holdings are stationary, then all long-term dynamics of banks' assets can be captured by only considering the loans.

Substituting (3) into (2), I can rewrite the above optimization problem as a Lagrangean:

$$\mathcal{L} = \max_{\{A_t, C_t\}_0^\infty} E_0 \sum_{t=1}^{\infty} \beta^t \left[ \phi(\Pi_t) + \lambda_t (C_t - \rho A_t) \right]$$

where  $\Pi_t = (1 + r_t)A_{t-1} - (1 + r_t^d)A_t + r_t^d C_t$ . The first order optimality conditions for an interior solution ( $C_t = \rho A_t \Rightarrow \lambda_t > 0$ )<sup>5</sup> are

$$(1 + (1 - \rho)r_t^d)\phi'(\Pi_t) = \beta E_t \left\{ \phi'(\Pi_{t+1})R_{t+1} \right\} \quad (5)$$

$$\lim_{T \rightarrow \infty} \left\{ \beta^T \phi'(\Pi_T)(1 + (1 - \rho)r_T^d) \right\} = 0 \quad (6)$$

The equation (5) equates the marginal "utility" of cost of funds raised to finance loans with the expected discounted marginal "utility" of returns - a standard intertemporal Euler condition. Equation (6) is the transversality condition which rules out explosive loan paths.

To continue with the analysis, I have to assume a specific functional form for the transformation function  $\phi(\cdot)$ . I choose to use a quadratic transformation because its computational friendliness (see Appendix B for a discussion of this point). That is, I assume:  $\phi(\Pi_t) = c_0 + c_1 \Pi_t - \frac{c_2}{2} \Pi_t^2$ . Defining  $R_t \equiv 1 + r_t$  and  $R_t^d \equiv 1 + (1 - \rho)r_t^d$  and normalizing  $c_2 = 1$ , Euler equation (5) can then be rewritten as a second order difference equation.

$$\beta E_t [A_{t+1} R_{t+1} R_{t+1}^d] - A_t [R_t^{d^2} + \beta E_t R_{t+1}^2] + A_{t-1} R_t R_t^d + \beta c_1 E_t R_{t+1} - c_1 R_t^d = 0 \quad (7)$$

### 3.2 Demand for credit

The demand side for credit in this problem is a version of the contracting problem in Bernanke, Gertler and Gilchrist (1999). At the beginning of period  $t$ , firm  $i$  ( $i \in \{1, \dots, N\}$ ) chooses the amount of capital that is used in production,  $K_t^i$  before a realization of a productivity shock  $\omega_t^i$ . The capital is homogeneous. The firm has no internal funds available and has to apply for a loan in order to finance its project<sup>6</sup>, and

<sup>5</sup>The corner solution occurs when the capital adequacy requirement does not bind, i.e. when the bank holds capital in excess of the regulatory requirement. The first order conditions in this case are:  $\phi'(\Pi_t) = 0$ ,  $(1 + (1 - \rho)r_t^d)\phi'(\Pi_t) = \beta E_t \{ \phi'(\Pi_{t+1})(1 + r_{t+1}) \}$  and  $C_t > \rho A_t$ . Because the primary reason of including the capital adequacy constraint into bank's decision problem is to motivate it to account for the risk, introducing risk-aversion in the way done in this paper reduces the need for this constraint. A more sophisticated model will account for this.

<sup>6</sup>This is a start-up assumption that allows me to analyze the model without the effects of the net worth. As I mention in the introduction, net worth fluctuations cause the credit cycles in models of financial accelerator, whereas here these can arise without the presence of internal financing.

pay a cost of  $c_t$  per unit of capital. After investment is made, the firm receives a return according to a linear production function  $1 + \omega_t^i K_t^i$ .

I assume that the productivity shock can be additively decomposed into idiosyncratic and aggregate components:  $\omega_t^i = \kappa^i + \mu_t$  where  $\kappa^i$  is a firm-specific shock ( $\kappa^i \sim \log N(0, \sigma_\kappa^2)$ ) and  $\mu_t$  is an aggregate shock  $\mu \sim \text{AR}(1)$  ( $\mu_t = \alpha_\mu \mu_{t-1} + \epsilon_{\mu_t}$ ,  $\epsilon_{\mu_t} \sim \log N(0, \sigma_\mu^2)$ ). The only characteristic distinguishing one firm from another is the idiosyncratic part  $\kappa^i$  of the productivity shock  $\omega_t^i$ .  $\kappa^i$  could be part of the private information set of a firm  $i$ . If its mean was positive,  $\kappa^i$  could be modeled as a proxy for the level of effort or skill entering the production process. This would motivate the screening mechanism by the bank and lead to the usual propagation through the net worth. Here I concentrate on how aggregate variables affect behavior of credit so I ignore this option and interpret  $\kappa^i$  as a shock which is i.i.d. and in public domain.

The firm is risk neutral and cares about profits, therefore its problem is simply:

$$\max_{K_t} (1 + \omega_t^i) K_t^i - (1 + c_t^i) K_t^i$$

for a given cost of capital (interest rate)  $c_t^i$ . The profit maximization implies

$$c_t^i = E\omega_t^i, \tag{8}$$

i.e., the firm acquires just enough capital for marginal cost to equal marginal revenue. Because of a CRS production function, firm's output – and therefore its capital level – is indeterminate.

### 3.3 Central bank

The central bank in this setup plays a simple role. Because I do not model households, the deposits needed to finance loans are supplied inelastically at a rate  $r^d$  which is controlled by the central bank. One can think of there being an interbank market for financing bank's liabilities, in which case the assumption of  $r^d$  being controlled by a central bank is more credible. The central bank in this model then also supplements the banks' capital market<sup>7</sup>. For simplicity, I assume that  $\{r_t^d\}$  follows an AR(1) process:  $r_t^d = \gamma_d r_{t-1}^d + \epsilon_{dt}$  where  $\epsilon_{dt} \sim \log N(0, \sigma_{Rd}^2) \forall t$ . This implies an AR(1) process for the gross deposit rate:  $R_t^d = (1 - \gamma_d) + \gamma_d R_{t-1}^d + \epsilon_{Rt} \forall t$ . Therefore,  $E_t R_{t+i}^d = (1 - \gamma_d) + \gamma_d E_t R_{t+i-1}^d = \dots = (1 - \gamma_d) + \gamma_d^i R_t^d \forall i \geq 0$ .

### 3.4 Market clearing

At the time when credit market decisions are made, none of the participants know a realization of the stochastic variables in period  $t$ . After the prices ( $c_t$ ) and quantities ( $A_t$ ) are determined, a new realization of a distribution of  $\{\omega_t\}$  is revealed.

<sup>7</sup>See Berka and Zimmermann (2001) on how market for banks' equity/capital affects the loan supply in an economy with heterogenous agents' setup.

Because the firms only care about the mean value of the productivity shock  $E\omega_t^i$ , banks will be able to perfectly diversify all idiosyncratic risk in their portfolio ex-ante. Therefore the only non-diversifiable risk facing the bank ex-ante is the aggregate risk – the realization of  $\mu_{t+1}$ .

The market clearing in the market for capital is implicit because of the indeterminate size of the firm  $i$ . The market clearing condition is:

$$\sum_{i=1}^N K_t^i = K_t^{agg} = A_t \forall t \quad (9)$$

Because the bank has no way of distinguishing between the firms, it charges all of them the same gross interest rate:  $c_t^i = c_t \forall i$ , which brings us back to the representative agent setup.

The simplistic demand side in this model implies that the behavior of capital in the economy is fully determined by the amount of credit banks supply. While this may be true of firms which have no access to other sources of external finance, overall this is an incredible assumption. Introduction of a non-degenerate demand side is left as a next step for future research. However, as long as the participants on the supply side of the capital market take risk into consideration, the results of this model will hold. To this extent, this channel of transmission will impact both large- and small-size firms.

To solve the model I need to specify the connection between expected returns on loans  $R_t$  and the interest rate  $c_t$  charged by the bank. Obviously, these are not identical. The rate of return on a loan is bounded from above by the interest rate and from below by zero, and its realization in this model depends on the realization of the aggregate productivity shock  $\omega$ . Therefore, expected value of  $R_{t+1}$  is a function of expected values of  $c_{t+1}$  and  $\omega_{t+1}$  (see Appendix C for some discussion).

Even though the distribution of productivity shocks affects the demand for capital in such a way that the average firm is able to pay off its loan ( $E_t(MPK_{t+1}) = E_t(1+c_{t+1})$ ), the expected return on a loan for bank will always be smaller than that:  $ER_{t+1} < E_t(MPK_{t+1})$ . This is because, from the point of view of the bank, the right half of the distribution collapses onto  $E_t c_{t+1}$  – the debt contract has no upside risk, only downside. For simplicity, I assume that this relationship is time-invariant:

$$E_t R_{t+1} = sE_t(1 + c_{t+1})$$

where  $s$  is a constant ( $s < 1$ ). I can now rewrite the distribution of expected returns on loans as a function of the distribution of aggregate shocks<sup>8</sup>:

$$E_t R_{t+1} = sE_t(1 + c_{t+1}) = sE_t(1 + \omega_{t+1}^i) = sE_t(1 + \mu_{t+1}) \quad (10)$$

$$\sigma_R^2 = \sigma_\mu^2 \quad (11)$$

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<sup>8</sup>With a more elaborate demand side, productivity growth and the level of capital would also influence the variance of expected returns.

The market for banks' capital is cleared implicitly due to the assumption of an infinitely elastic supply of capital by the central bank. In the case of an interior solution, equation (4) implies that

$$C_t = \rho A_t, \quad (12)$$

and so the market clearing level of credit determines the path of capital. The deposit market is also cleared implicitly due to an infinitely elastic demand for deposits by the central bank at a going rate  $r_t^d$ . Equations (3) and (4) determine the market-clearing level of  $D_t$  as a function of  $A_t$ :

$$D_t = (1 - \rho)A_t. \quad (13)$$

### 3.5 Equilibrium

The equilibrium is a process of  $\{r_t, r_t^d, c_t, K_t^i, A_t, C_t, D_t, \mu_t, \kappa_t^i, \sigma_A^2, \sigma_R^2, \sigma_{R^d}^2, \sigma_\mu^2, \sigma_\kappa^2\}$ , such that banks solve their optimization problem (equation (7)), firm's maximize their profits (equation (8)) and markets for credit, deposits and banks' capital clear (equations (9) through (13)).

## 4 Solution

### 4.1 Deterministic case

I first solve for the deterministic case. The the second order difference equation (7) can be solved by factorizing the polynomial and then, if the eigenvalues are well behaved, inverting one of the factors. This method is only applicable when the coefficients depend only on variables within the current (time  $t$ ) information set. For illustration, in deterministic case I let  $R_t \equiv 1 + r_t = 1 + r \forall t$  and  $R_t^d \equiv 1 + r_t^d = 1 + r^d \forall t$ . The Euler equation can then be rewritten into an equilibrium process for capital (see Appendix A):

$$A_t = \frac{R^d}{R\beta} A_{t-1} + \frac{c_1}{c_2} \frac{R\beta - R^d}{(R - R^d)R\beta} \quad (14)$$

Because the AR(1) coefficient of this rule is less than one, the steady state of lending is  $A^{ss} = \frac{c_1}{c_2} \frac{1}{R - R^d}$ . Due to the problem's simplicity, banks will optimally aim to stabilize their profits by removing any variability that may arrive from the changes in the rate of return  $R$  or the funding rate  $R^d$  ( $\Pi^{ss} = (R - R^d)A^{ss} = \frac{c_1}{c_2}$ ). Note that the steady state as well as the transitional dynamics depend on the ratio of the parameters  $c_1$  and  $c_2$ . Therefore the specifics of the dynamics as well as the steady state will depend on the exact functional form of the transformation function, and in turn on the degree of risk aversion as well as higher derivatives. The second term on the RHS of equation (14) which is a sum of an infinite profit margins will disappear in the special case when  $\phi(\cdot) = \log(\cdot)$ , because the intertemporal substitution and income effects offset each other and there is no advantage from a gradual adjustment to a steady state (i.e.,

following a shock, bank will instantly move to a new steady state). Even though  $A^{ss}$  declines in the "interest rate margin"  $R - R^d$ , the transition to the steady state using calibrated values for returns (see section 5 below) takes an exceptionally long time<sup>9</sup>.

## 4.2 Stochastic case – Exact solution method

The optimality condition (7) above contains expectations of a product of future realizations of variables. The expectations depend on all moments of the variables included in the product. While in the deterministic case only means of the variables stayed in this equation, stochastic case will obviously retain the other moments as well.

First I need to separate the products inside of the expectations operator. In order to do so, I will limit my analysis to a family of processes for the rate of return and the deposit interest rate that will turn out to be the actual realizations in the equilibrium. I assume that these variables are log normally distributed<sup>10</sup> with a constant variance, which will turn out to be correct in equilibrium. This allows us to rewrite (7) as:

$$(B^{-2} + \frac{\Psi_t}{b_t}B^{-1} + \frac{\nu_t}{b_t})E_t A_{t-1} = \frac{z_t}{b_t} \quad (15)$$

where  $\Psi_t$ ,  $b_t$ ,  $\nu_t$  and  $z_t$  are functions of current and future distribution of  $R$  and  $R^d$  and  $B$  is a backshift operator (see Appendix A for details). In order to solve this difference equation, the parameters  $\Psi_t$ ,  $\nu_t$  and  $b_t$  must only depend on variables in the information set at time  $t$ . I have to make an assumption about the way the expectations are formed and this process has to turn out to be the equilibrium one.

Making use of the equilibrium conditions (10) and (11), we see that the rate of return on loans assumes an AR(1) process<sup>11</sup>:  $R_{t+1} = s(1 + \mu_{t+1}) = s(1 + \alpha_\mu \mu_t + \epsilon_\mu) = s((1 - \alpha_\mu) + \alpha_\mu(1 + \mu_t) + \epsilon_\mu) = s(1 - \alpha_\mu) + \alpha_\mu R_t + s\epsilon_\mu$ . Therefore,  $E_t R_{t+i} = s^i((1 - \alpha_\mu^i) + \alpha_\mu^i R_t) \forall i \geq 1$ . Also,  $\sigma_R^2 = var[\log(R_{t+1})] = var[\log(s((1 - \alpha_\mu) + \alpha_\mu(1 + \mu_t) + \epsilon_\mu))] = var[\log s\epsilon_{\mu,t}] = var[\log \epsilon_{\mu,t}] = \sigma_\mu^2$ .<sup>12</sup>

The assumptions on the expectation-formation processes of the two rates allow us

<sup>9</sup> With a starting value  $A_0 = 0.5A^{ss}$ , the transition half way to the steady state takes about 100 years.

<sup>10</sup> Obstfeld and Rogoff (1999) show that in a simple stochastic general equilibrium model, endogenous variables have a log normal distribution if the exogenous variables are jointly log normally distributed.

<sup>11</sup> A simpler alternative is to assume that the rates of return on loans and deposit rates are expected to remain unchanged, either as a random walk process or even more simply equal to a constant. All the results carry through qualitatively in both of these cases, but I do not report them as the assumption on rate processes are less realistic. As I discuss in the section ?? below, the results of the model are robust to a wide range of AR(.) processes.

<sup>12</sup> Upon inversion, present values of  $R_t$  and  $R_t^d$  will be used for extrapolation into an infinite future, which may seem to be unrealistic. However, there is striking evidence that banks indeed do extrapolate current conditions into future when calculating their expected payoffs. Unfortunately, this evidence is confidential.

to rewrite coefficients  $\Psi_t$  and  $z_t$  in the equation (18) as:

$$\Psi_t = -\frac{R_t^{d^2} + \beta(\alpha_\mu R_t + s(1 - \alpha_\mu))^2 \exp \sigma_R^2}{(\gamma_d R_t^d + 1 - \gamma_d)(\alpha_\mu R_t + s(1 - \alpha_\mu))}, \quad z_t = \frac{c_1 R_t^d - \beta E_t R_{t+1}}{c_2 E_t R_{t+1}^d E_t R_{t+1}}$$

$$\nu_t = \frac{R_t R_t^d}{(\gamma_d R_t^d + 1 - \gamma_d)(\alpha_\mu R_t + s(1 - \alpha_\mu))}$$

I can now solve for the optimal credit supply rule in the presence of uncertainty:

$$A_t = \lambda_1 A_{t-1} + \lambda_1 \sum_{i=0}^{\infty} (\lambda_1 \beta \exp(\Omega))^i \left[ \frac{c_1 \beta E_t R_{t+i+1} - E_t R_{t+i}^d}{c_2 E_t R_{t+i+1}^d E_t R_{t+i+1}} \right]$$

where

$$\lambda_{1,2} = \frac{-\Psi_t \mp \sqrt{\Psi_t^2 - 4\beta \exp(\Omega)\nu_t}}{2\beta \exp(\Omega)}$$

and so  $\lambda_1 < 1 < \lambda_2$ <sup>13</sup>. On assumption that  $E_t R_{t+1} = R$  and  $E_t R_{t+1}^d = R^d \forall t$ , this can be further simplified into:

$$A_t = \lambda_1 A_{t-1} + \frac{1}{\beta \exp(\Omega)} \frac{1}{\lambda_2 - 1} \left[ \frac{c_1}{c_2} \left( \frac{\beta R - R^d}{R^d R} \right) \right] \quad (16)$$

which implies a steady state value of

$$A^{ss} = \frac{1}{\beta \exp(\Omega)} \frac{1}{(1 - \lambda_1)(\lambda_2 - 1)} \left[ \frac{c_1}{c_2} \left( \frac{\beta R - R^d}{R^d R} \right) \right].$$

Section 6.2 below discusses in detail the behavior of the credit in both deterministic and stochastic cases.

## 5 Calibration

In this section I explain the calibration of parameters and exogenous variables. The parameters of the concave function  $\phi(\cdot)$  control the level of risk-aversion. I normalize all parameters  $c_i = 1$ ,  $i = \{1, 2, 3\}$  which poses no threat to a partial equilibrium analysis. The capital adequacy parameter  $\rho$  is set to 0.08 in accordance with the "old" Basle Accord. The time discount factor  $\beta$  is computed as a monthly equivalent of the standard annual value 0.96, therefore  $\beta = 0.9966$ .

The rate of return on loans is hard to come by, not to mention an expected rate of return. In this model, rate of returns on loans assumes the distribution of productivity shocks, and should be therefore calibrated using that type of data. I should calibrate

<sup>13</sup>Note that the variance terms move both eigenvalues further from unity relative to their values in a deterministic case.

the productivity shocks using my linear production function. This production function is quite unrealistic and using it to estimate productivity shocks will only amplify this. Use of productivity estimates from other production functions (e.g. Cobb-Douglas) will lead to another inconsistency. I avoid this issue for now and calibrate the returns process using monthly data on the Canadian chartered bank administered prime business interest rate. This rate is not equal to the average rate of return on loans, but it can be argued that it is a reasonable proxy. There are two factors which shift the realized as well as expected rate of return on loans away from the prime business lending rate, yet they work in opposite directions and therefore may partly cancel each other. On the one hand, firm's bankruptcies lower the rate of return below the prime business lending rate. On the other hand, many businesses are charged interest rates above the prime rate. The second issue applies mainly to the smaller businesses without an access to the alternative funding sources which are in the focus of this paper. It is not clear how is the variance of the expected rate of return on loans affected by this. Appendix C outlines these issues in a more structured way.

On the deposit side, I use monthly data on the Canadian non-chequable savings deposit rate for a typical chartered bank. The starting point for the data is 1968, and the ending point is chosen to be 1992. The selection of the endpoint is due to an apparent structural break in the series (see Figure 3) in the early 1990's after which the rate dropped to 0.5%. This raises the variance of the deposit rate far above that of the prime rate. I do not think that the uncertainty of the deposit rate increased above that of its loan portfolio, hence the cutoff date in 1992. The mean of the deposit rate series for a period of 1968 until 1992 is 7.06%, after 1992 it is 0.39%. Then means of the interest rate series are 10.8% for the prime rate and 6.62% for the deposit rates, with monthly equivalents of 0.0086 and 0.0055, respectively.

The variances of the rates of return and deposit are taken from the same series as the rates. The problem with using the prime business loan rate to calibrate the expected rate of return on a loan is more obvious when determining the variances: the risk of loan repayment is likely rising through an economic downturn, even if the rate remains unchanged or adjusts with a lag. For the purposes of the model I need variances of transformed interest rate series  $\log(1+c_t)$  and  $\log(1+(1-\rho)r_t^d)$ . Therefore the calibrated values are  $\sigma_{\tilde{R}}^2 = 0.0008186$  and  $\sigma_{\tilde{R}^d}^2 = 0.0006024$  (variances of raw series are 0.1021 and 0.0874, respectively)<sup>14</sup>. The coefficient  $\Omega$  in the equation (18) above contains variances of  $\tilde{A}$ , and all the covariances. Because the supplied credit is an endogenous variable and a result of the optimization, I do not use the calibrated values, i.e.  $\sigma_{\tilde{A}}^2 = 0$ . The same is the case for  $\sigma_{\tilde{A}\tilde{R}}$  and  $\sigma_{\tilde{R}^d\tilde{R}}$ . The distribution of  $\mu$  is not calibrated.

The two remaining parameters  $\alpha_\mu$  and  $\gamma_d$  are estimated using the same interest rate series which are used for calibrating their means and variances. An OLS regression

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<sup>14</sup> What we are calibrating is the variance of the expected rate of return on loans. Using the restrictions imposed by our equilibrium AR(1) process on  $R$ , this turns out to equal the  $var(\log(1+r_t))$ :  $var(E_t \log R_{t+1}) = var(\log(\gamma R_t + 1 - \gamma)) = var(\log(\gamma R_t)) = var(\log(\gamma) + \log(R_t)) = var(\log(\gamma)) + var(\log(R_t)) = var(\log(R_t)) = var(\log(1 + r_t))$ .

yields parameter values  $\hat{\alpha}_\mu = 0.998$  and  $\hat{\gamma}_d = 0.996$ .

## 6 Results

In this section I first show the properties of the calibrated credit processes. In order to obtain more insights about the dimensions through which monetary policy affects credit in this model, I run policy simulations in various environments.

### 6.1 Behavior of credit in a deterministic case

A rise in the rate of return has two effects: (a) it lowers the steady state level of credit supplied and (b) it makes the transition to the new steady state faster. With an equal relative distance from their steady states, banks will adjust faster when their rate of return on loans increases. As Figure 4 illustrates, the level of credit supplied is higher for a significant time period when the level of loans at the time of the shock is sufficiently below the steady state level. An increase in the annual rate of return from 10.8% (the mean of Canada's chartered banks' business prime rate for the period of 1970 to 1992) to 12.8% increases the speed of convergence<sup>15</sup> by a factor of 2.61 (for an increase of 1 percentage point, the speed of convergence rises by a factor of 1.88). If (as I argue above) the steady state  $A^{ss}$  is somewhat arbitrary, then the main insight of the optimal credit supply rule (14) is that in the deterministic perfect foresight case, banks will want to supply credit faster when, for a given deposit rate and capital adequacy ratio, the rate of return rises. The changes in the level of deposit rate will have opposite effects on the behavior of loan volume.

### 6.2 Behavior of credit in a stochastic case

First I analyze the *properties of the steady state* level of credit. The stochastic  $A^{ss}$  is lower than the deterministic one. Intuitively, this is because the variance of returns and deposits around their long-run means motivates the bank to lower its exposure by decreasing the supply of credit. It is instructive to see the effects on the  $A^{ss}$  of changes in  $R^d$  for the simulations. An increase in  $R^d$  lowers the wedge between the two eigenvalues ( $\lambda_1$  increases while  $\lambda_2$  declines) and at the same time increases the infinitely discounted interest rate margin  $\frac{c_1}{c_2} \left( \frac{\beta R_t - R_t^d}{R_t^d \hat{R}_t} \right)$ , all of which decreases the steady state level of credit  $A^{ss}$ . Changes in  $R$  have an opposite effects on  $A^{ss}$ , yet with a more pronounced magnitude<sup>16</sup>.

<sup>15</sup>Relative convergence refers to the amount of time needed to overcome half of the distance from the starting point to the steady state (for a proportionally same starting point). The relative  $\beta$ -convergence refers to the time it takes for the credit supply to rise by 1 unit (of the half-distance).

<sup>16</sup>An elasticity of  $\lambda_i$  w.r.t.  $R$  is 12 times larger in an absolute value than the elasticity of  $\lambda_i$  w.r.t.  $R^d$ .

An increase in the variance of the rate of return  $\sigma_R^2$  lowers  $\lambda_1$  and increases  $\lambda_2$ , decreasing the  $A^{ss}$  as a result. Changes in the variance of the deposit rate,  $\sigma_{R^d}^2$ , have an opposite effect but are significantly less pronounced due to a partly offsetting impact on levels of  $\lambda_i$ s and  $b$ <sup>17</sup>. Numerically obtained comparative statics of the impacts of variances on  $A^{ss}$  evaluated at the benchmark calibration show that the magnitude of the response of  $A^{ss}$  to a unit change in  $\sigma_R^2$  is 30 times larger in absolute value than a response to a unit change in  $\sigma_{R^d}^2$ .

Secondly I analyze the behavior of the credit supply along the *transition paths*. The transitions between steady states take a significant amount of time and may therefore be more relevant for characterizing the observed behavior of credit than the steady state analysis. Variation in the rates or their variances affects the speed of transition through  $\lambda$ s and through the discounted value of the interest rate margin. These two effects work in opposite directions. An increase in  $R$  (decline in  $R^d$ ) decreases the speed of convergence through its impact on  $\lambda_i$  but increases the speed of convergence because of the rise in the interest rate margin. Using the calibrated values from the section 5, a rise in  $R$  by 2 percentage points from it's mean increases the relative speed of  $\beta$ -convergence by a factor of 4.36. Similarly, a decline in the deposit rate by 0.25 of a percentage point from it's mean raises the relative speed of  $\beta$ -convergence by a factor of 1.46<sup>18</sup>. This occurs because the higher rate of return increases bank's steady state profits, while during the transition (due to a higher growth rate of credit), the profits are temporarily lower than what they would be otherwise (see figure 5d). This motivates the bank to move through this transition period at a higher speed. Note that the impact on the transition dynamics is stronger now than in the deterministic case.

Because the variance in the rate of return in this setup is an inefficiency, an increase in  $\sigma_R^2$  lowers the rate of growth of credit supplied. The impact of changes in variances on  $\lambda_i$ s and on the discounted interest rate margin affect the transition in an identical direction, not like the changes in the interest rate as we saw above. Therefore the temporary drop in profits needed to support higher growth (and steady state level) of credit motivates the bank to overcome this period faster than following a rate shock (see figure 5d). Moreover, the elasticities of  $\lambda_i$ s w.r.t.  $\sigma_R^2$  are 10 times larger than the respective elasticities w.r.t.  $R$  (the factors for  $\sigma_{R^d}^2$  and  $R^d$  are 3.2 for  $\lambda_1$  and 6.5 for  $\lambda_2$ ). Therefore, changes in variances of the rates may affect growth rates of credit more strongly than changes in the expected rate of return. For example, an increase in the  $\sigma_R^2$  from 0.0008 to 0.0009 lowers the relative speed of  $\beta$ -convergence by a factor of 0.54.

I have learnt several things about the credit behavior in a stochastic environment. First, the uncertainty about the rate of return (for an unchanged level of return) results in an "inefficiently" low steady-state level of credit. Second, increases in the rate of return (or declines in the deposit rate) have stronger effects in the presence of

<sup>17</sup>The elasticity of  $\lambda_1$  with respect to  $\sigma_R^2$  is 67 times larger in absolute value than the elasticity of  $\lambda_1$  with respect to  $\sigma_{R^d}^2$ . For  $\lambda_2$  the ratio is 23.

<sup>18</sup>For a starting point equal to 1/3 of the pre-shock  $A^{ss}$ .

uncertainty. They increase the steady state level of credit more dramatically than when no uncertainty is present, and also lead to a higher growth rate/speed of convergence than in the deterministic case. Third, changes in the variance (uncertainty) of the rate of return or the deposit rate have a significant impact on both the steady state and the speed of convergence. Indeed, their effect is larger than the effect of changes in the expected rates. Therefore, changes in the uncertainty that occur during the business cycle lead to significant fluctuations in the amounts of credit supplied, even when the expected rates of return and deposit remain unchanged.

### 6.3 Graphing the credit

Figure 4 shows the simulated series for credit for a benchmark situation in the deterministic case. Figure 5 plots a similar situation for the stochastic case. The introduction of the uncertainty significantly lowers the steady state. Starting in both cases from 1/3 of the steady state level, the relative speed of  $\beta$ -convergence is computed to be higher by a factor of 2.57 in the stochastic case compared to the deterministic case. Both figures plot the simulated behavior following a return rate increase by 2 percentage points from the benchmark case. Figures 6a and 6b compare the effects of changes in the deposit interest rates in the presence of average variance of returns to a  $\sigma_R^2$  higher by a factor of 1.087, starting from the steady state (Figure 7 makes a similar comparison in case the bank uses an approximate credit supply rule which is described in Appendix D). At a higher uncertainty, the relative speed of  $\beta$ -convergence is computed to be higher by a factor of 1.26 as bank is more motivated to get to a new steady state (which is lower than the starting one). Because in our model the deposit rate is controlled by a central bank, we can think of a simple policy experiment, whereas the monetary action can be used to eliminate the negative effect of a higher level of uncertainty. Figure 6d shows how a gap between steady state levels of credit in "good" and "bad" states (see figure 6c) can be wiped out with a 25 b.p. monetary easing.

To further clarify the contribution of relative effects of changes in variances and changes in the monetary policy instrument, figure 8a shows the impulse response function of credit following a monetary easing of 25 b.p. at different levels of uncertainty. This figure attempts to be a summary representation of all four graphs in figure 6a and refers to the same thought experiment: a monetary policy easing is offsetting the negative effect due to the increase in the variance of the expected rate of return  $\sigma_R^2$  by a factor of 1.087. The graph shows us that at the higher level of uncertainty, an 25 b.p. monetary easing will have a relatively stronger effect on the level of credit, as well as on the speed of adjustment. Note, however, that as we saw in figure 6, the starting steady state is lower at a higher level of uncertainty. In other words, these impulse response functions illustrate the higher responsiveness of the credit to monetary policy during the times of higher uncertainty over future return rates on loans (and vice versa in the case of lower uncertainty). Figure 8b shows that this finding is more pronounced when banks use an approximate credit supply rule.

## 6.4 Simulations with time-varying uncertainty

In this subsection I enrich the above analysis by considering a process for the variance  $\sigma_{\bar{R}}^2$ . The variance of the rate of return on loans – even the business prime rate proxy – varies over time (Figure 9 plots a 5-year and 1-year rolling window of the variance of the prime rate). The model predicts that the behavior of variance of returns is determined in equilibrium by the behavior of the variance of aggregate productivity shocks<sup>19</sup> as well as the marginal product of capital. To assume that the variance of the returns on loans is procyclical is therefore not illogical: it simply states that the uncertainty (variance) about the future productivity growth rate is highest at the peak of the business cycle<sup>20</sup>.

I therefore construct a series for  $\sigma_{\bar{R}}^2$  with a mean identical to that of the data, and with the level fluctuating in the range of mean  $\pm 20\%$ . I chose the phase of the cycle to match the average length of the NBER postwar peacetime business cycles (1945–1991) of 53 months with the functional form  $\sin(\cdot)$ <sup>21</sup>. Figure 10a plots a discretized version of these series. The simulated series for  $\sigma_{\bar{R}}^2$  implies a process for  $\lambda_i$ s which governs the steady state credit level as well as the transitional dynamics of the system.

I perform the following experiment. I assume that banks observe a new realization of  $\sigma_{\bar{R}}^2$  from the underlying process every six months. Extrapolating this value of the variance (the expected rate of return remains unchanged) into the infinite future, they come up with a linear credit supply rule which then governs the market clearing amount of credit in the economy. After 6 months a new realization of  $\sigma_{\bar{R}}^2$  is revealed and the banks proceed with a new optimal credit supply rule. Due to the fluctuating variance, and because of the sensitivity of the eigenvalues to  $\sigma_{\bar{R}}^2$ , the steady state level of credit exhibits fluctuations (see figure 10b). A smaller wedge between the two eigenvalues brings the state closer to the first-best. Because banks extrapolate the status quo into the future, the steady state to which loans converge rises. This implies credit behavior as shown in figure 10c and 10d. The level of credit fluctuates with an opposite phase than the uncertainty. Bringing this result to the data is left for future research (figure 1 shows raw and HP-detrended general loan series for Canada). Fluctuations in  $\sigma_{\bar{R}}^2$  also change the growth rates throughout the cycle, as we have seen in the analysis above (see figure 5c).

It is clear that the under reasonable assumptions, our model exhibits credit cycles. Although the source of these shocks can be interpreted as identical to the source of the shocks to the net worth in the financial accelerator literature, the propagation channel is different. Without any asymmetric information among the banks and firms, the *supply* of credit varies as risk-averse banks extrapolate the current conditions into the

<sup>19</sup>A standard non-degenerate demand side would also imply a connection between the *growth rate* of the aggregate productivity to impact the variance of rate of returns.

<sup>20</sup>In December 2000, the forecasts of the annual growth rate of real U.S. GDP in 2001 ranged from -4% to +5%.

<sup>21</sup>Specifically, I consider a variance process  $\sigma_{\bar{R}t}^2 = \hat{\sigma}_{\bar{R}}^2 + \sin(t\pi 0.24)/6024$  where  $t$  is the time period,  $\pi = 3.1416$  and  $\hat{\sigma}_{\bar{R}}^2$  is the estimated value of 0.0008186.

future. This channel affects identically small and large firms.

This section showed more clearly the importance of using a general equilibrium setup (or, at minimum, a non-degenerate demand side) for modeling of the behavior of credit. The growth rate and variance of the aggregate productivity measure usually play an important role in GE models, but never has an impact on the supply of credit. Incorporating a risk-averse banking sector into a GE model will allow for this additional linkage because of the effect productivity growth rate and its variance have on the expected rate of return on loans supplied by the banks.

## 7 Credit crunch and implications for monetary policy

The cyclical credit behavior is usually modeled as a result of a negative shock which (a) lowers the rate of return on the project of any given firm and (b) decreases firm's net worth. The first effect increases the probability of default of any borrower while the second lowers the post-default payoff to the lender. The credit crunch is then partly a result of a decline in demand due to increasing "tightness" of the firm's credit constraint, whose existence is justified by an assumed presence of asymmetric information.

The model presented here shows that credit cycles can emerge in a world without private information between the agents. Because the net worth plays no role in the determination of the demand for credit, credit levels fluctuate only due to changes in the perceived uncertainty about the future loan return. The assumption of asymmetric information can be added into the model to enrich dynamics, but it is not necessary for generating credit fluctuations. The model is able to explain why credit volume as well as its growth rate may decline at the peak of the business cycle when the uncertainty of the future average economic performance, measured by the variance of the productivity shocks in this setup, is (supposedly) highest. This would colloquially be considered a case of a "credit crunch". However, as I stressed in the introduction, the definition of a credit crunch is model-contingent, not axiomatic. Even though banks in this model cut their total amount of extended credit following an increase in an uncertainty, this is an optimal decision. Indeed, if the risk-aversion is imposed on the banking sector by benevolent regulators (the "society"), then the simulated "credit crunch" is a socially optimal outcome. In other words, there is no credit crunch in this model, only an optimally fluctuating credit supply.

The role of monetary policy can not be analyzed normatively in the partial equilibrium framework I present here. However, there are a few implications for monetary policy that I can draw from the insights obtained from this model. As we have seen, an increase in  $\sigma_R^2$  can be offset by a monetary easing (decline in  $R^d$ ). This suggests that monetary policy *can* be used to prevent the credit decline that would otherwise occur. Because the decline in the steady state following a negative shock is accompanied by a slowdown in the growth rate of credit, the monetary easing increases the steady state level as well as growth of credit.

The second important insight is that the effects of the monetary policy are cycle-

dependent. The impact of a change of  $R^d$  on  $A^{ss}$  as well as the transition dynamics depend on  $\sigma_R^2$  which depends on the variance of the aggregate productivity shocks. When the variance of expected returns is low,  $A^{ss}$  is more responsive to the changes in  $R^d$ . Figure 11c displays how the "efficiency" of monetary policy changes over the simulated cycle. Figure 12 traces the effectiveness of the monetary policy as the variance of expected returns increases. It shows that with increasing uncertainty of future returns, the potency of monetary policy declines. This argument would justify a relatively bolder monetary policy actions when the rates of return (and the variance of productivity shocks) is higher than normal in order to have the same impact as during more peaceful times.

To summarize, this model implies that monetary easing may be warranted even without a change in the rate of return, purely due to the result of higher variance of expected returns. Further, monetary actions have asymmetric effects depending on the predictability of the economic environment. In times of high variance of expected returns on loans (at the peak of the business cycle), the magnitude of the move by the central bank will have to be larger in order to have the same affect as during an early stage of an expansion (when the uncertainty of future returns is lower).

## 8 Conclusion and future research

This paper specifies a quantitative framework for analyzing the behavior of credit in a stochastic environment. Non-linearity of the banks' profit function gives rise to a forward looking credit supply rule. The behavior of the mean as well as the variance of the expected rate of return influences the level of credit in the economy. Indeed, credit level as well as the growth rate is sensitive not only to the expected rate of return bank is to receive from a loan, but also the the variance of this rate. In uncertain times, when variance of the rate of return increases, banks supply less credit, even for an unchanged mean rate of return. Because the variance of the rate of return on loans is in equilibrium determined by the variance of aggregate productivity shocks, this model explains the cyclical behavior of credit during the business cycle without the assumption of asymmetric information. In turn, it can explain credit contractions at the peak of a business cycle as well as to provide an additional channel for monetary policy shocks to have an effect on real economic activity. Due to the lack of assumption of asymmetric information, the model predicts that small and large firms will be affected identically by the monetary policy (or aggregate productivity) shocks.

In a positive way, this paper shows when and how can monetary policy be used to offset shocks to the environment. Monetary policy can be *preemptive* to offset banks' inefficiently low credit supply during the times of higher variance of productivity shocks. Moreover, monetary policy actions are *asymmetric* as the impact of a unitary deposit rate change is smaller at higher rates of uncertainty. In that sense, this paper provides a justification for a bolder policy action at times when uncertainty about the future returns is higher.

The work that needs to be done can be roughly divided into two categories. On the modeling side, the demand side should be introduced in a non-degenerate way. A representative agent methodology would be easier to handle due to computational intensity. Introducing an asymmetric information assumption may be considered to enrich the dynamics with that implied by the standard financial accelerator models. Thus enriched, the credit market should be nested in a general equilibrium setup<sup>22</sup>, which would allow a more insightful analysis of model's predications. Because the framework we use distinguishes between current and expected rates of return as well as the deposit rates, it is well suited for analyzing the effects the timing and announcement effects of the monetary policy. The opportunities to analyze model's predictions empirically will be much larger once it is set in a general equilibrium framework.

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<sup>22</sup>A standard example of this is Bernanke, Gertler and Gilchrist (1999).

## A Appendix: Derivations of the optimal credit supply rule in the deterministic case

### A.1 Deterministic case

The second order equation can then be written as:

$$(B^{-2} + \frac{\Psi}{\beta}B^{-1} + \frac{1}{\beta})E_t A_{t-1} = \frac{z}{\beta} \quad (17)$$

where

$$\Psi = -\frac{R^{d2} + \beta R^2}{RR^d} \quad z = \frac{c_1 R^d - \beta R}{c_2 RR^d}$$

and  $B$  is a backshift operator. Equation (17) can be expressed as  $(\lambda_1 - B^{-1})(\lambda_2 - B^{-1})E_t A_{t-1} = z/\beta$  where  $\lambda_1$  and  $\lambda_2$  are the eigenvalues which in this case equal  $\lambda_1 = \frac{R^d}{\beta R}$  and  $\lambda_2 = \frac{R}{R^d}$ , respectively<sup>23</sup>. Operating on both sides of the polynomial with  $(1 - \frac{B^{-1}}{\lambda_2})^{-1}$  causes the RHS be an expected infinite discounted sum where the discount factor is a function of one of the eigenvalues. This yields the solution:

$$A_t = \lambda_1 A_{t-1} + \lambda_1 \sum_{i=0}^{\infty} (\lambda_1 \beta)^i \left[ \frac{c_1}{c_2} \left( \frac{\beta R - R^d}{R^d R} \right) \right]$$

This equation describes the linear decision rule for the level of loans the bank wants to supply, expressed as a function of last period's loans and an infinitely discounted sum of future profit margins. Because the interest rate terms on the RHS are constant, we can further reduce the optimal lending rule to equation (14) in section 3.1.

### A.2 Stochastic case

The assumption of lognormality of  $A$ ,  $R$  and  $R^d$  allows us to rewrite the optimality condition (7) as:

$$(B^{-2} + \frac{\Psi_t}{b_t}B^{-1} + \frac{1}{b_t})E_t A_{t-1} = \frac{z_t}{b_t} \quad (18)$$

where

$$\Psi_t = -\frac{R_t^{d2} + \beta(E_t R_{t+1})^2 \exp \sigma_{\tilde{R}}^2}{R_t^d E_t R_{t+1}}, \quad b_t = \beta \exp(\Omega), \quad z_t = \frac{c_1 R_t^d - \beta E_t R_{t+1}}{c_2 R_t^d E_t R_{t+1}}$$

$$\Omega = \frac{1}{2}\sigma_A^2 + \frac{1}{2}\sigma_{\tilde{R}}^2 + \frac{1}{2}\sigma_{\tilde{R}^d}^2 + \sigma_{\tilde{A}\tilde{R}} + \sigma_{\tilde{A}\tilde{R}^d} + \sigma_{\tilde{R}\tilde{R}^d}$$

In deriving the above set of equations, we use the fact that if  $X$  and  $Y$  are log normally distributed then  $E(XY) = E(X)E(Y) \exp \sigma_{\tilde{X}\tilde{Y}}^2$  ( $\tilde{X}$  denotes  $\log(X)$ ,  $\sigma_{\tilde{X}}^2$  is variance of  $X$  and  $\sigma_{\tilde{X}\tilde{Y}}$  is covariance of  $X$  and  $Y$ ).

<sup>23</sup>  $\lambda_1$  and  $\lambda_2$  solve a system:  $\lambda_1 \lambda_2 = 1/\beta$ ,  $|\lambda_1 + \lambda_2| = -\Psi/\beta$ . Without a loss of generality, we assume that  $|\lambda_1| < 1 < |\lambda_2|$ . One of the eigenvalues has to be less than one in absolute value for the problem to have a solution, therefore in our case we have to assume  $R^d < R\beta$ .

## B Appendix: On the assumption of risk-averse banks

The assumption of risk-averse banks can be viewed in various ways. Technically, we are introducing a non-linearity into bank's optimization problem. This is important because a linear function generally does not have a maximum and the solution is degenerate. Banks are not thought of as having a "production function" like firms, therefore they rarely play any role in a dynamic setup. Introduction of a non-linearity is essential for a model which intends to have banks play a non-degenerate role. This is what the "utility" function in this model does. The fact that we do not assign it a specific interpretation is only of a secondary importance. The interpretation would only assign a meaning to the driving force behind the risk-averse behavior, and as we have seen in section 3.1, there are at least 6 of these. We do not feel competent to do this, and prefer to stress the fact that no matter what is a leading cause of a non-linearity in the bank's profit function, it *always* leads to the qualitatively identical results to those in this model.

The choice of a particular functional form for  $\phi(\cdot)$  also is not crucial for obtaining the results. The first law of arithmetic states that any function can be decomposed as a sum of linear, quadratic, cubic, ... etc. functions which are respectively weighted by a first, second, third, ... etc. derivatives of the original function (this is often used in Taylor series approximation). The quadratic function we chose can therefore be viewed as an approximation of any concave function (concavity is assured by the negative sign before the quadratic term). Different values for parameters  $c_0$ ,  $c_1$  and  $c_2$  would approximate different concave functions, which is why we can normalize them conveniently to 1.

To conclude, the assumption of a concave transformation is a generalization of a non-linearity, and can therefore comfortably nest any of their justifications. Rather than focusing on the specific cause, we show that *any* of them will lead to the results.

## C Appendix: Lending rate versus the rate of return on loans

The rate of return on loans is not identical to the lending rate. Therefore the variance of the expected rate of return on loans is also not identical to the variance of the lending rate. Although we use the prime business rate for calibration, it is instructive to clarify the connection between these two for future research.

The rate of return on loans and its variance can be expressed respectively as:

$$\begin{aligned} R = \min\{1 + r^I, 1 + r^L\} &= 1 + r^I \text{ iff } r^I < r^L \\ &= 1 + r^L \text{ iff } r^I > r^L \\ \text{var}(R) &= \text{var}(1 + r^I) \text{ iff } r^I < r^L \\ &= \text{var}(1 + r^L) \text{ iff } r^I > r^L. \end{aligned}$$

And consequently the expected rate of return on loans and expected variance of this rate are, respectively:

$$\begin{aligned} ER &= p_B(1 + r^I) + (1 - p_B)(1 + r^L) \\ \text{var}(E(R)) &= p_B^2 \text{var}(1 + r^I) + (1 - p_B)^2 \text{var}(1 + r^L). \end{aligned}$$

Where  $p_B = \text{prob}(r^I < r^L)$  can be interpreted as a probability of bankruptcy. This implies that as long as the probability of bankruptcy is not zero, the calibration we use is incorrect. Although  $p_B$  could be calibrated with the firm-level data available, this does not fit easily into our representative agent setup. More easily adoptible but a lot less perfect proxy is a proportion of bad in total loans in banks' portfolio.  $r^I$  can be calibrated with the return on investment data.

The second issue is that the lending rate  $r^L$  charged by the bank is not always equal to the prime rate.  $r^L$  exceeds the prime rate for firms that are perceived to be more risky which suggest that  $r^{L^i} = r^L(p_B^i)$ , i.e., lending rate to the firm is a function of the perceived probability of bankruptcy. This further complicates the above relationships which can now be rewritten as:

$$\begin{aligned} ER^i &= p_B^i(1 + r^I) + (1 - p_B^i)(1 + r^L(p_B^i)) \\ \text{var}(E(R^i)) &= (p_B^i)^2 \text{var}(1 + r^I) + (1 - p_B^i)^2 \text{var}(1 + r^L(p_B^i)). \end{aligned}$$

Note that these hold at the levels of individual firms and it is unclear how to aggregate them. For now, we ignore this problem altogether.

## D Appendix: Approximate credit supply rule

Coming soon...

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## E Figures

Figure 1: Log of Canadian general loan series, crude and HP-filtered

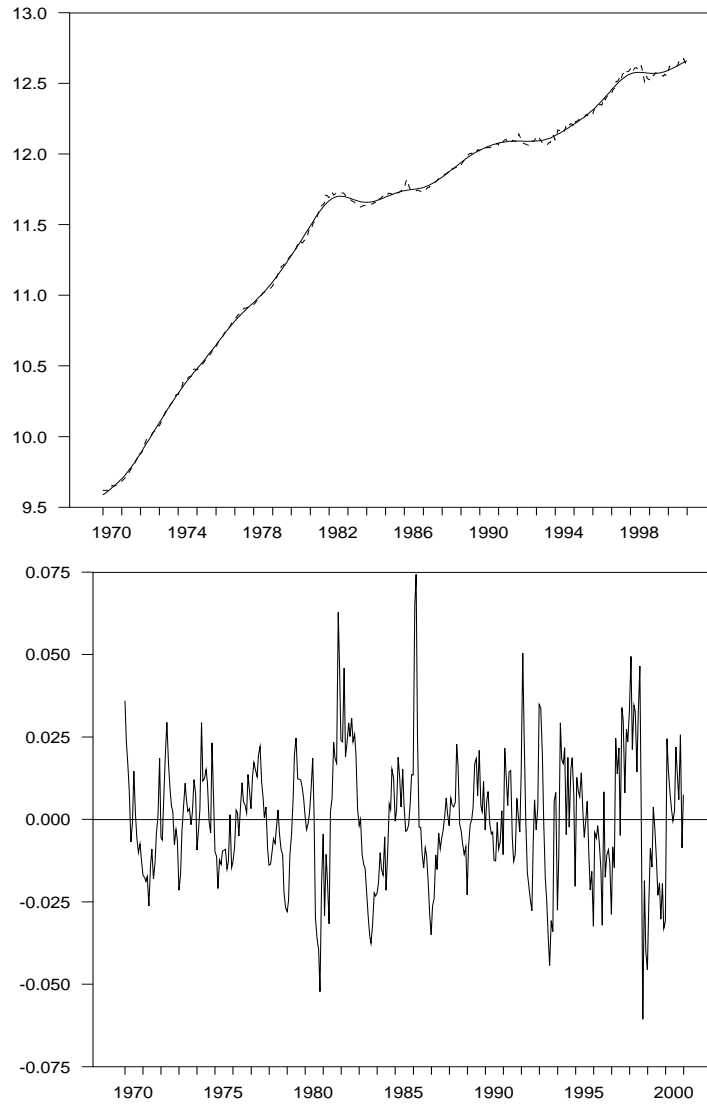


Figure 2: Prime business rate as an AR(1) process

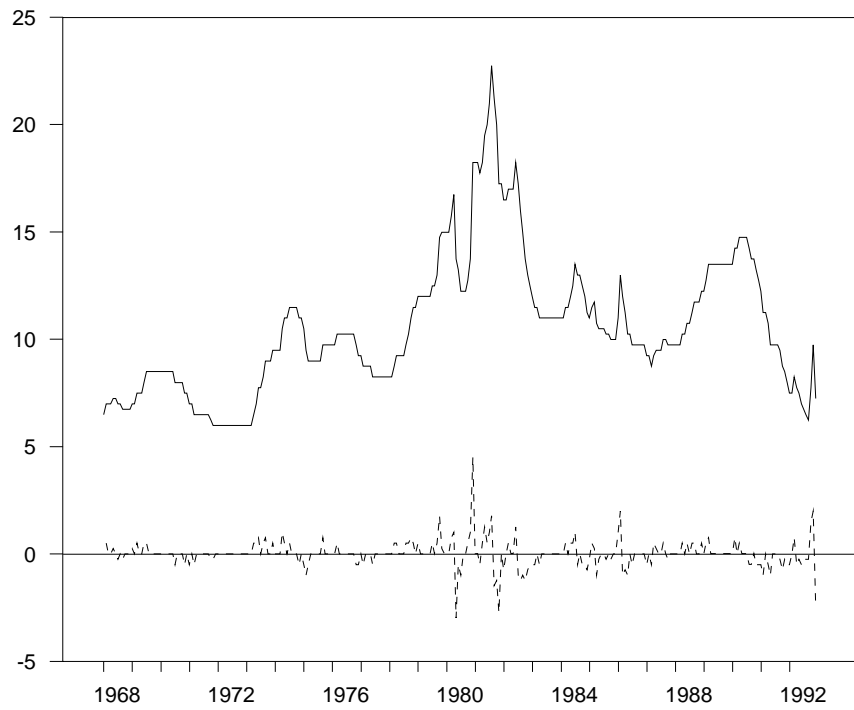
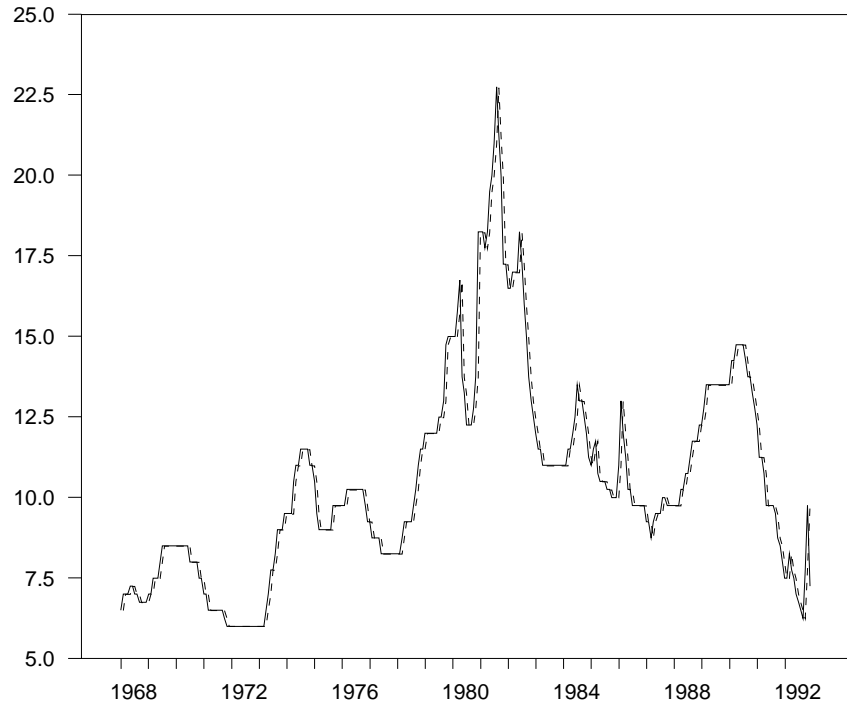


Figure 3: Why I chose 1992 as a cutoff date

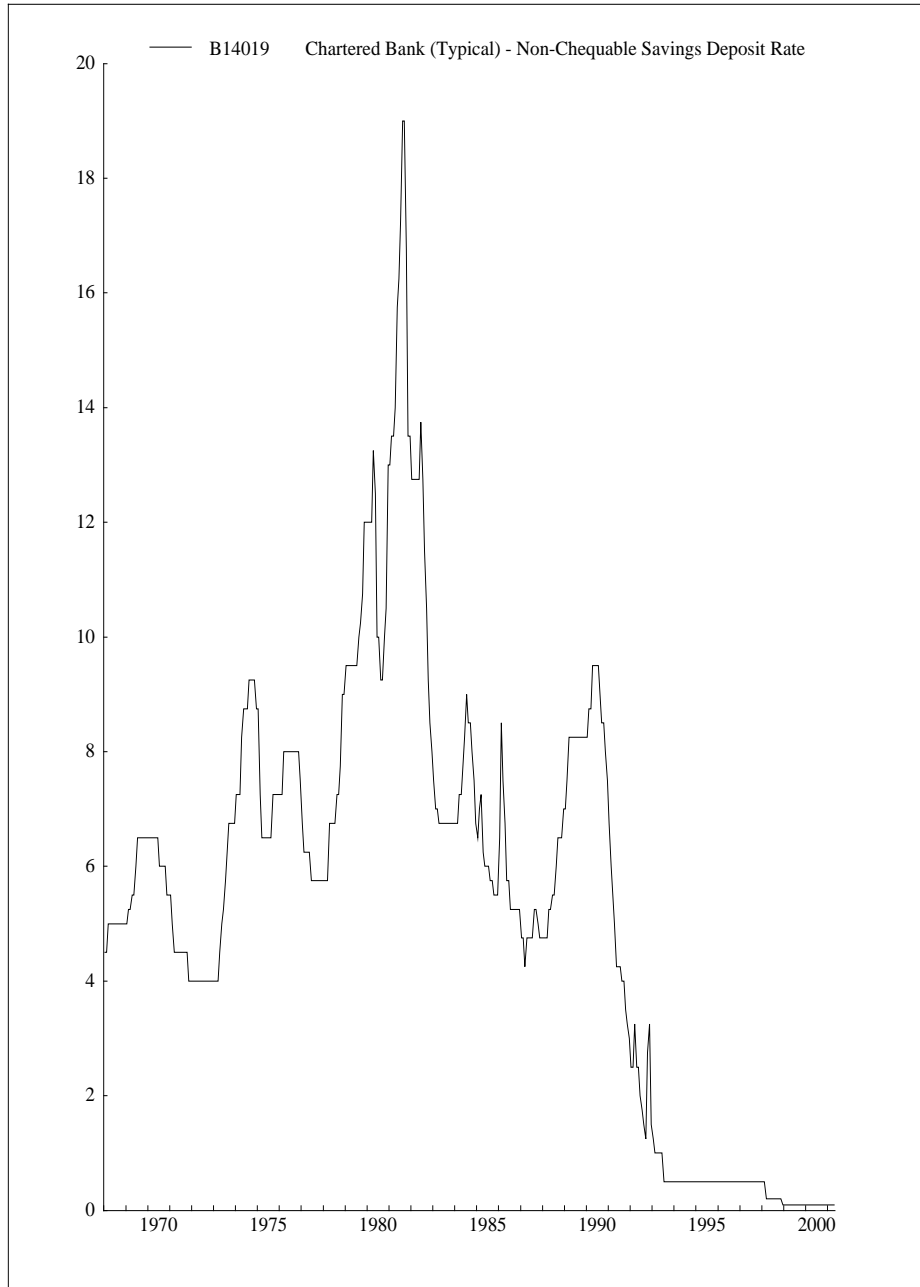
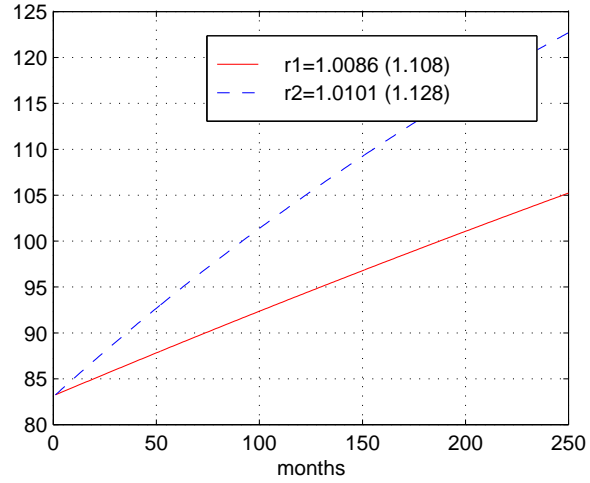
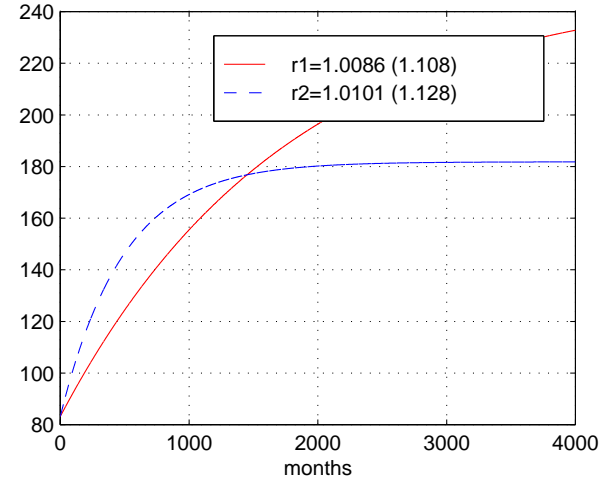


Figure 4: Credit Supply in a deterministic case

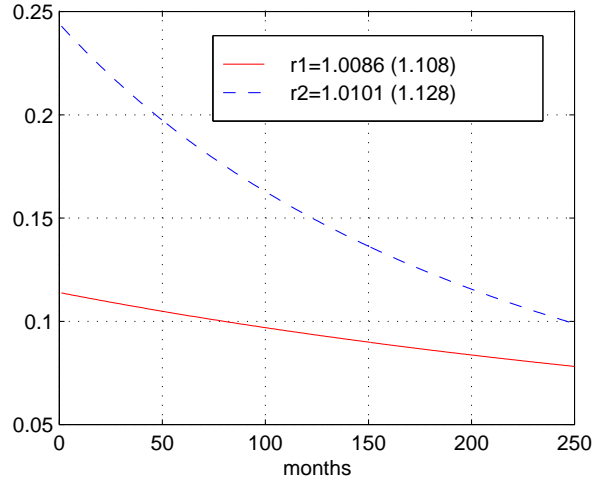
a) Higher return on loans rises the speed of convergence ( $A_0 = 1/3 A_{SS}$ )



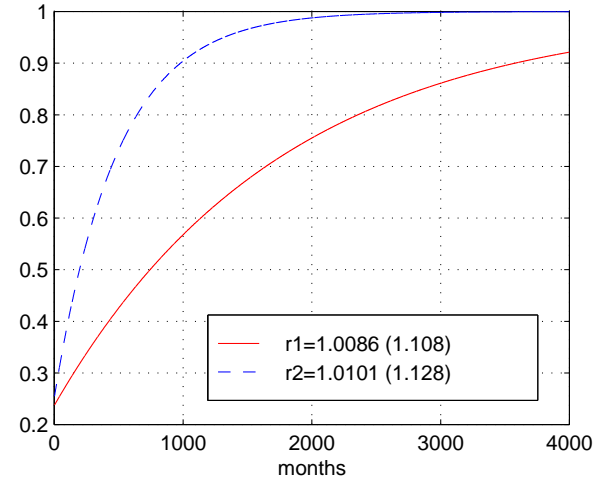
b) Higher return on loans lowers SS loans but speeds up transition



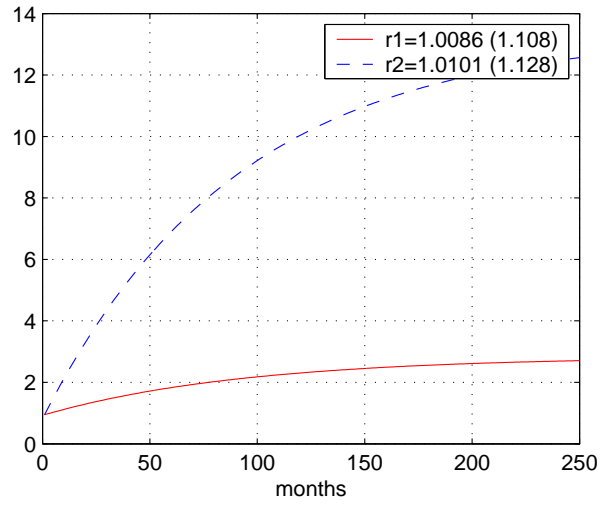
c) Growth rates of credit



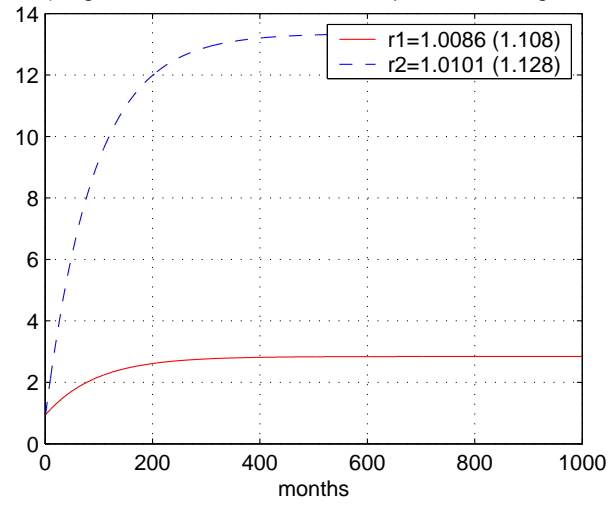
d) Momentary bank profits



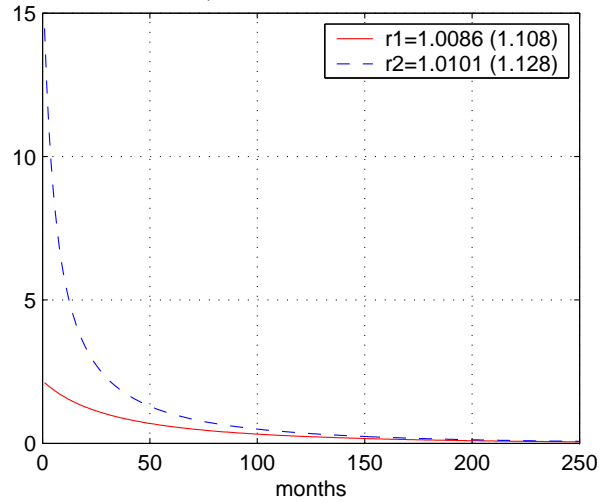
a) Higher return on loans raises steady state credit ( $A_0 = 1/3 A_{SS}$ )



b) Higher return on loans increases speed of convergence



c) Growth rates of credit



d) Momentary bank profits

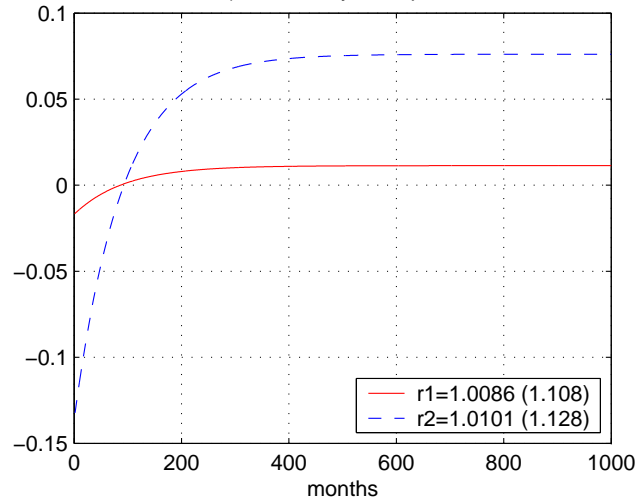


Figure 5: Credit Supply in a stochastic case

Figure 6: What happens after a  $\sigma_R^2$ -shock with and without monetary policy

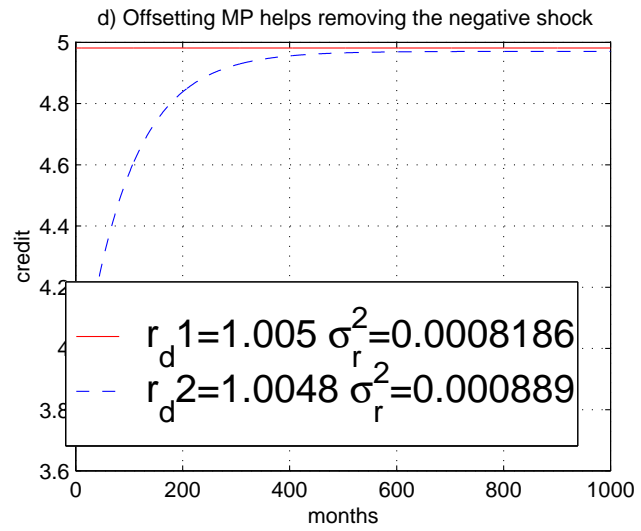
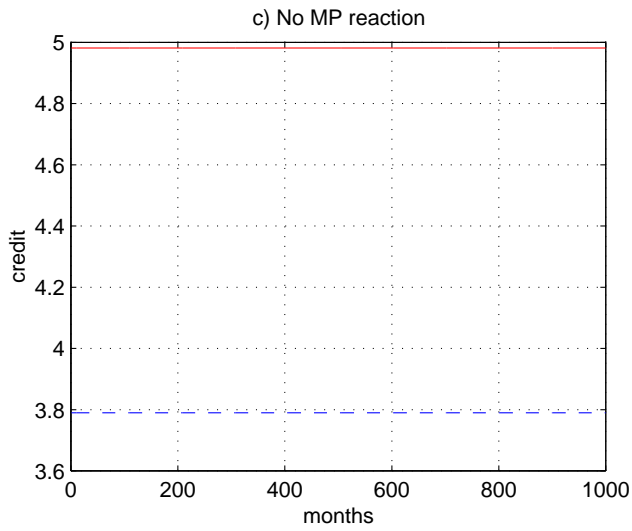
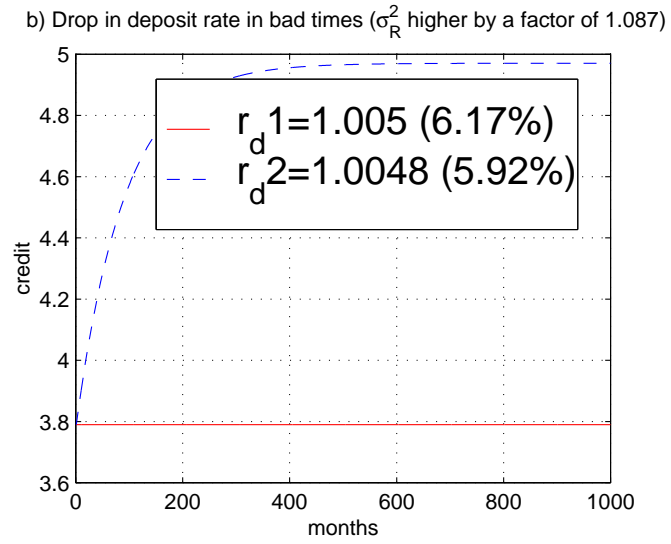
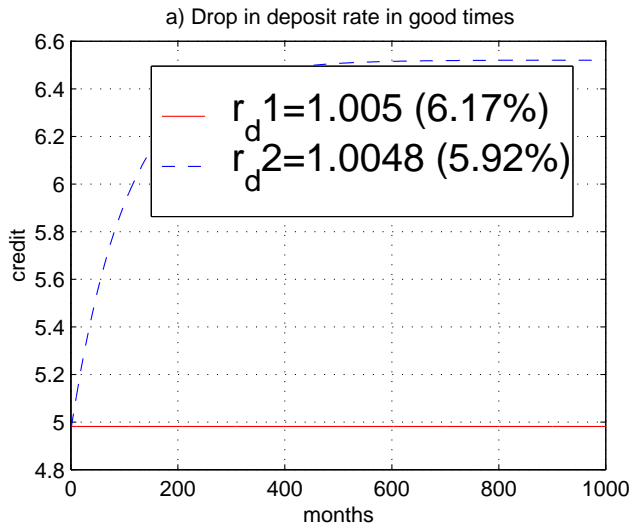


Figure 7: What happens after a  $\sigma_R^2$ -shock with and without monetary policy - using an APPROXIMATE credit supply rule

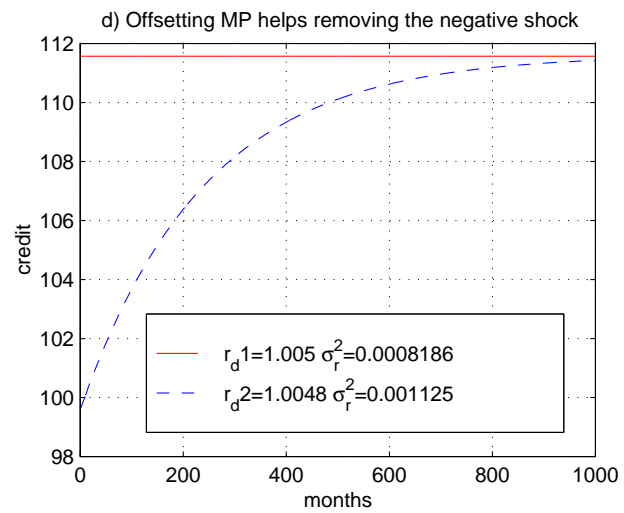
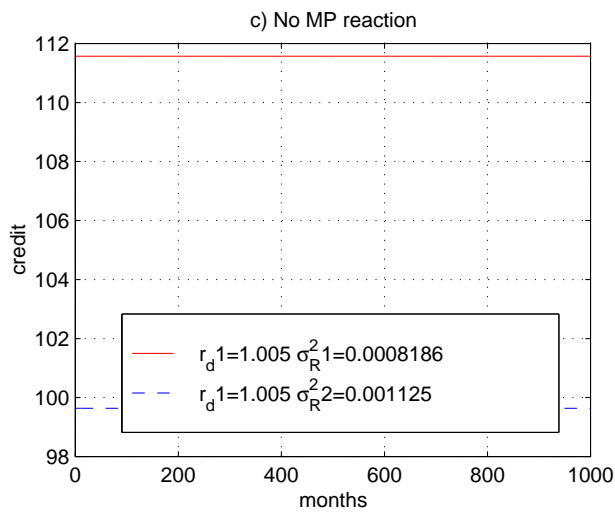
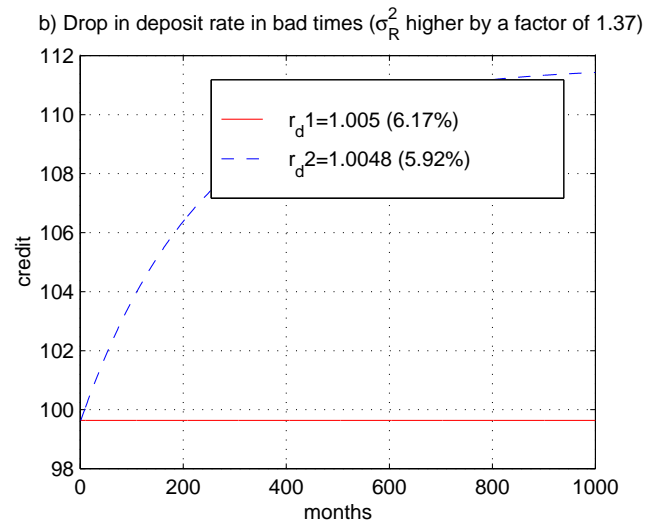
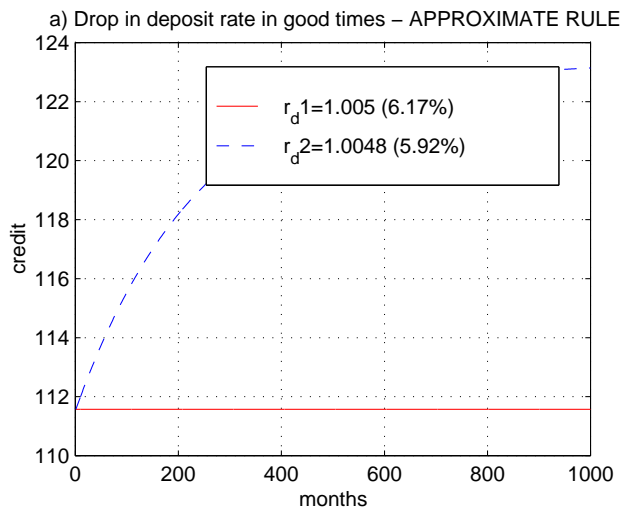


Figure 8: Impulse responses at an average and higher rate of uncertainty at complete and APPROXIMATE credit supply rule

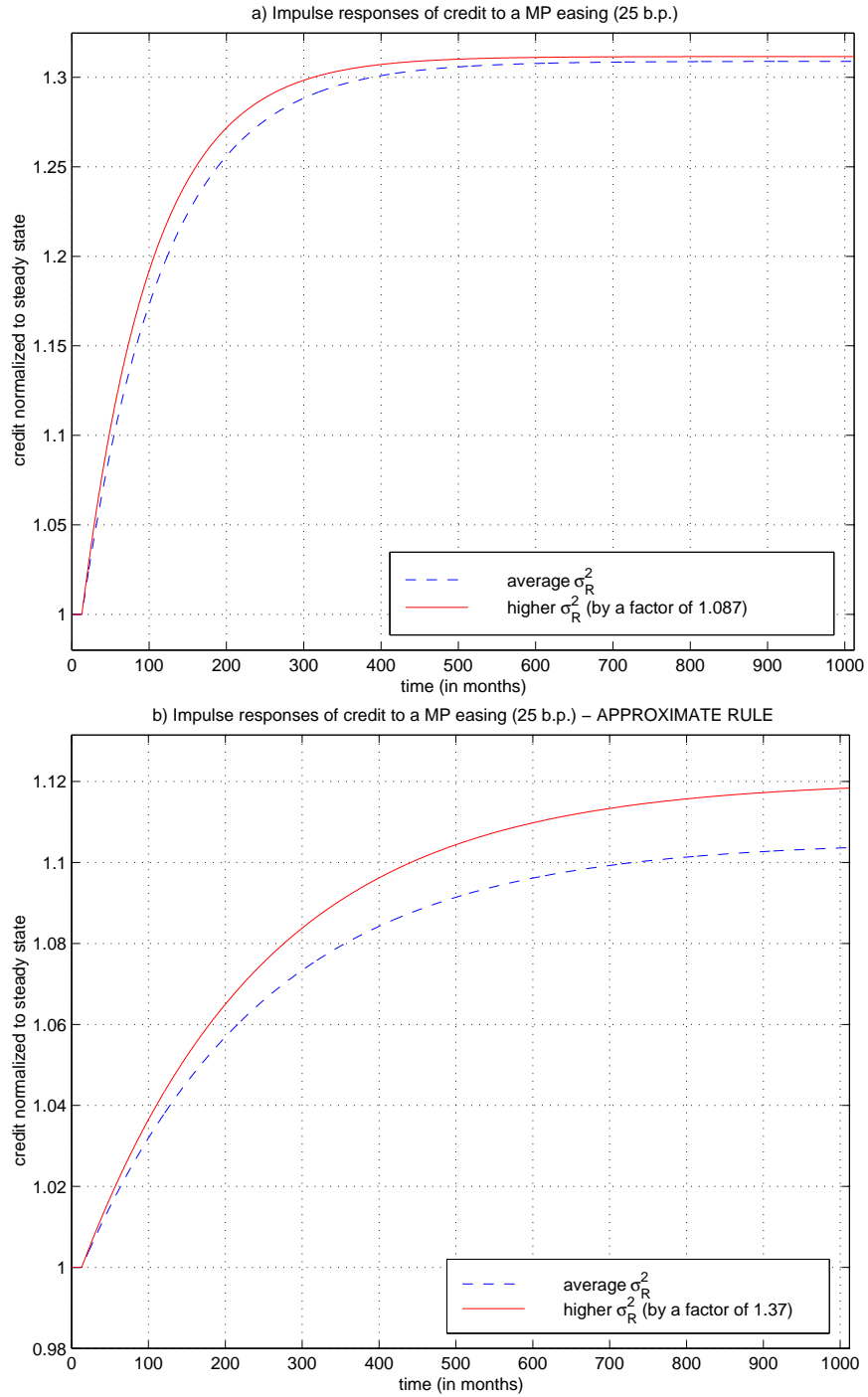


Figure 9: Variance of R varies fluctuates over time

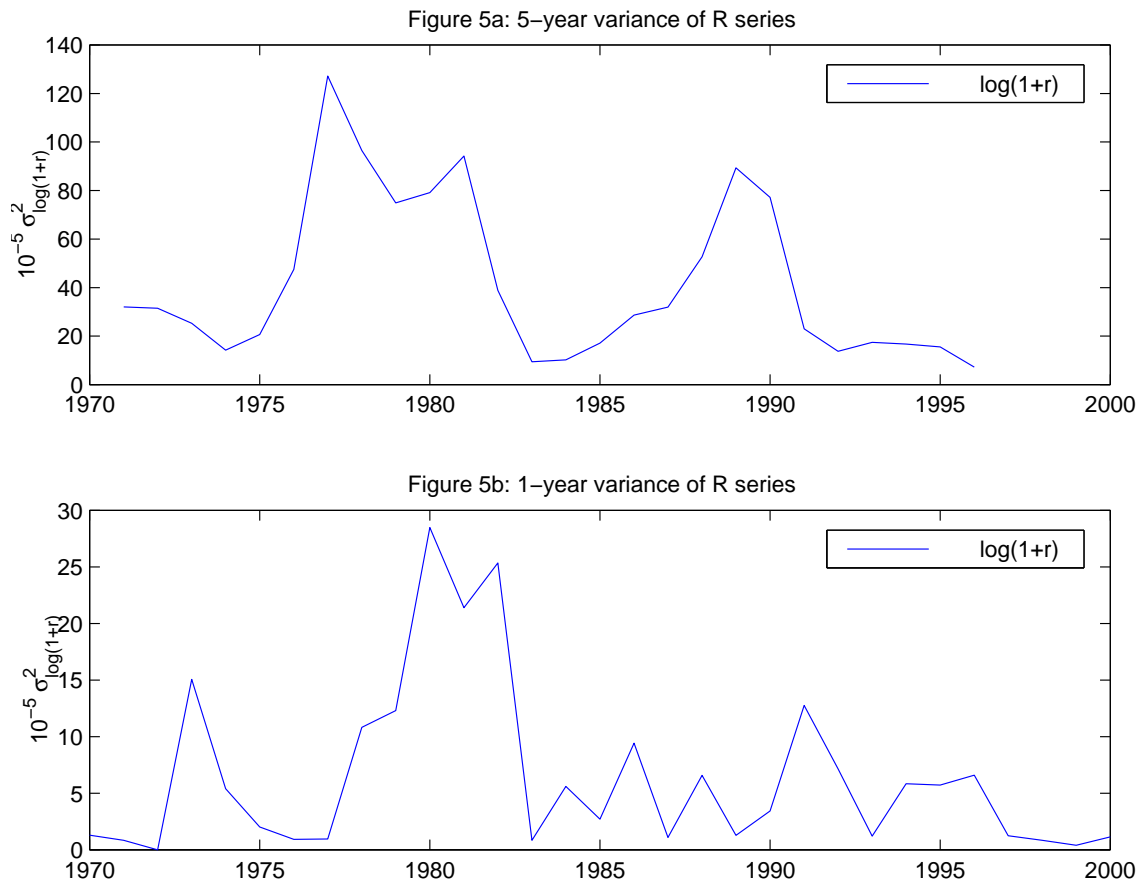


Figure 10: Simulation with a cyclical variance

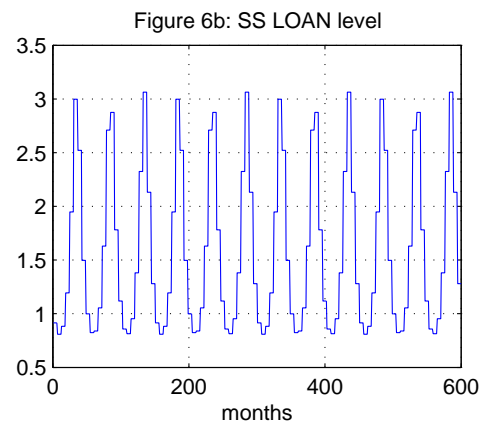
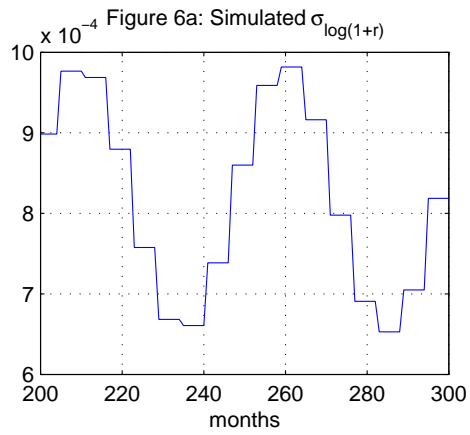


Figure 6c: LOANS (solid) AND UNCERTAINTY (dashed, scaled by 100)

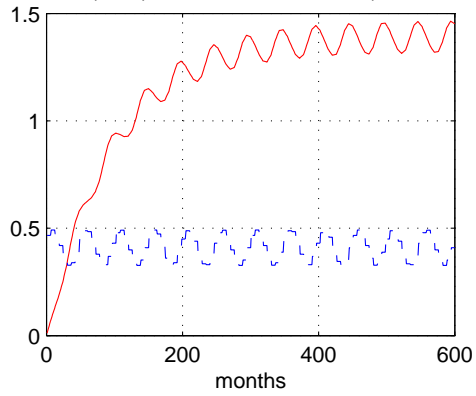


Figure 6d: Credit

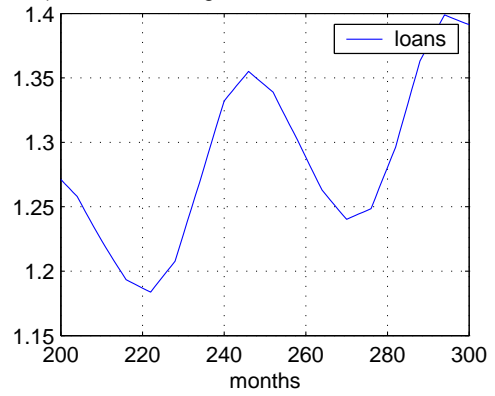


Figure 11: Asymmetric effects of monetary policy throughout the cycle

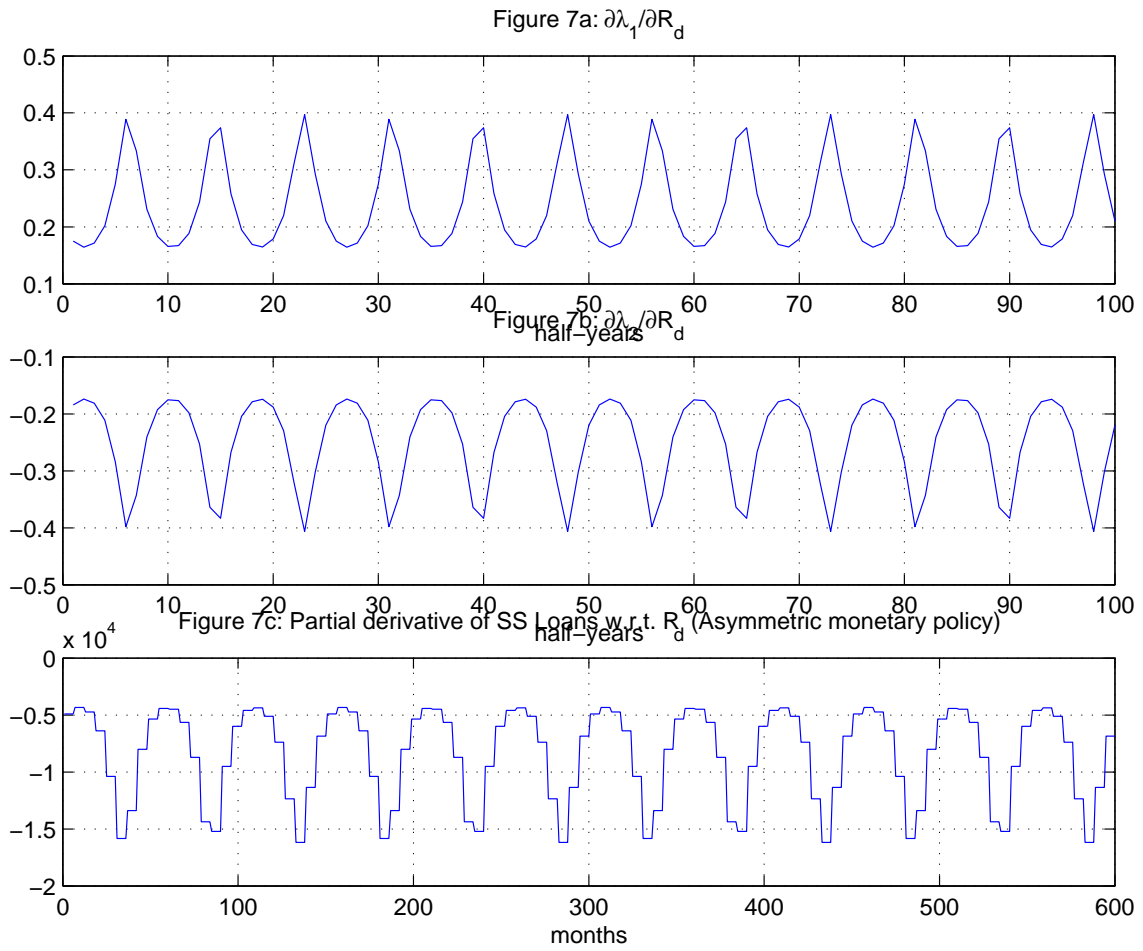


Figure 12: Asymmetric effects of monetary policy

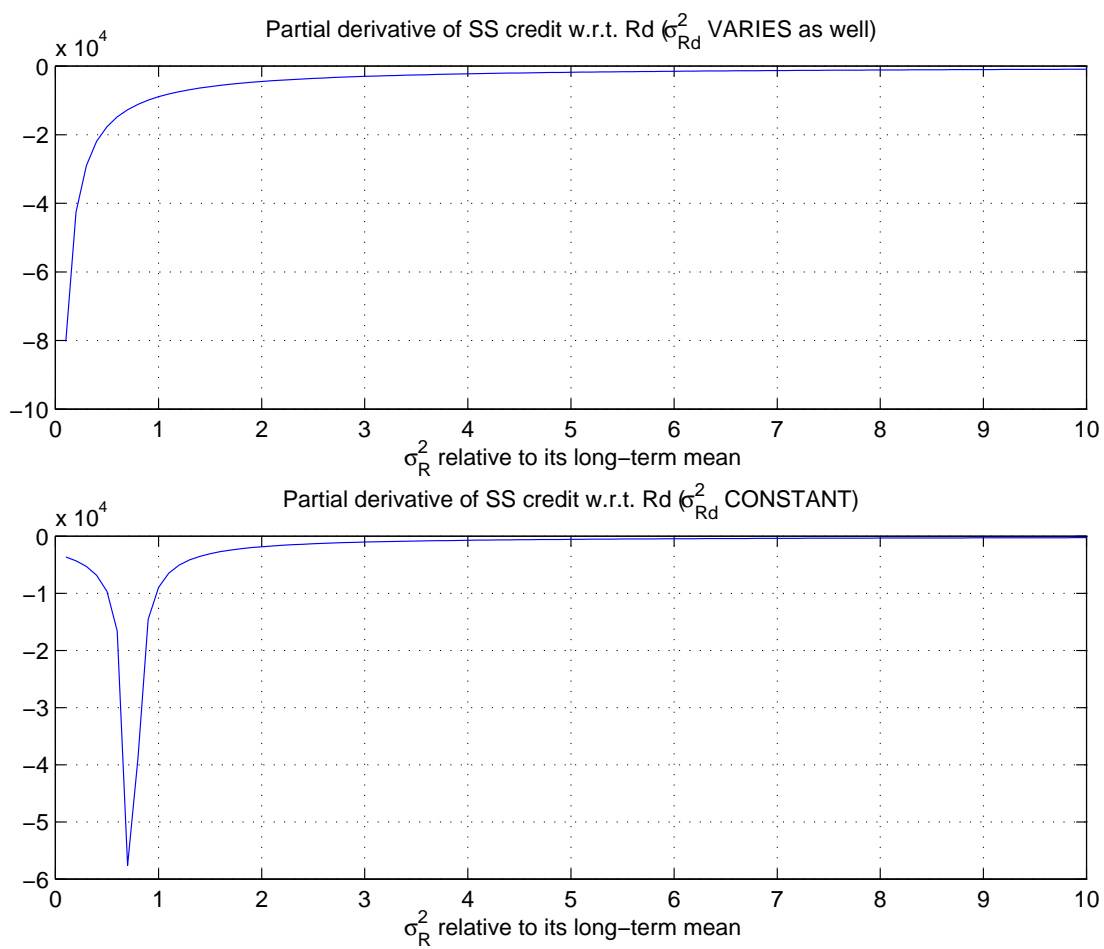


Figure 13: Robustness analysis of AR(1) processes of rate of return on loans R

Robustness analysis of credit supply rule for different AR(1) processes for  $r_t$  with time-varying  $\sigma_R^2$

