Inference for Day-to-Day Dynamic Traffic Models

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Joint Work with Katharina Parry (PhD Candidate)

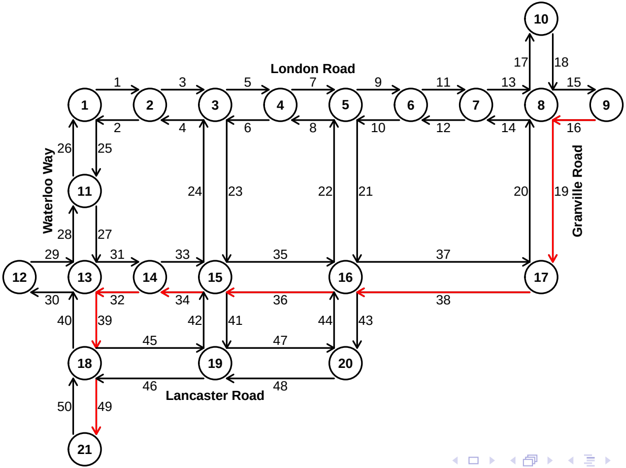
Field work!?

Network Based Transport Models

- Describe flow of vehicles through network.
- Used for a variety of traffic planning and control purposes.
- Come in many flavours:
  - Static or dynamic
  - Equilibrium; mesoscopic; micro-simulation
- Heavily parameterized
- Defined at micro level, but macro properties of interest

An Example Network

From city of Leicester, U.K.



Model Parameters

- Travel demand parameters critical
- Examples:
  - Origin-destination (OD) rates of traffic flow
  - Route choice probabilities
- May model demand as function of travel costs

Data for Model Fitting

- **Link Counts**
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  - GPS etc. (only fraction of vehicles)

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Will focus on fitting with link count data only.

The Indeterminacy Problem

- Many model parameters relate to route flows.
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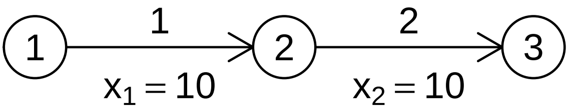
Highly underdetermined linear system:

$$\boldsymbol{x} = A\boldsymbol{y}$$

where $A = (a_{i\ell})$ is routing matrix for which

$$a_{i\ell} = \begin{cases} 1 & \text{if link } i \text{ is part of path } \ell \\ 0 & \text{otherwise.} \end{cases}$$

Toy Example



$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_{12} \\ y_{13} \\ y_{23} \end{bmatrix} = A\boldsymbol{y}$$

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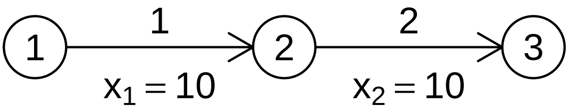

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Conditional on $\boldsymbol{x} = (10, 10)^\mathsf{T}$, latent route flows might be $\boldsymbol{y} = (0, 10, 0)^\mathsf{T}$ or $\boldsymbol{y} = (5, 5, 5)^\mathsf{T}$, or ...

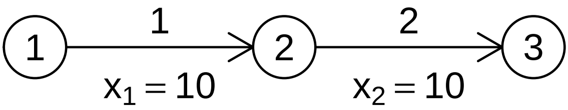
Likelihood Functions

- Likelihood (for static, discrete traffic models):

$$L(\boldsymbol{\psi}) = \sum_{\boldsymbol{y}} f(\boldsymbol{x}|\boldsymbol{y},\boldsymbol{\psi}) f(\boldsymbol{y}|\boldsymbol{\psi}) = \sum_{\boldsymbol{y}\in\mathcal{Y}(\boldsymbol{x})} f(\boldsymbol{y}|\boldsymbol{\psi})$$

  - $\mathcal{Y}(\boldsymbol{x}) = \{\boldsymbol{y} : \boldsymbol{x} = A\boldsymbol{y}\}$ is feasible route set
  - $\boldsymbol{\psi}$ model parameters

Toy Example Redux



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$$\begin{aligned} \mathcal{Y}(\boldsymbol{x}) &= \{(y_{12}, y_{13}, y_{23})^{\mathsf{T}} : y_{12} + y_{13} = x_1, y_{23} + y_{13} = x_2\} \\ &= \{(0, 10, 0)^{\mathsf{T}}, (1, 9, 1)^{\mathsf{T}}, \ldots, (10, 0, 10)^{T}\}. \end{aligned}$$

Computation of the Likelihood

- Recall
$$L(\boldsymbol{\psi}) = \sum_{\boldsymbol{y} \in \mathcal{Y}(\boldsymbol{x})} f(\boldsymbol{y}|\boldsymbol{\psi})$$
- Full evaluation of $L(\boldsymbol{\psi})$ requires all solutions to $\boldsymbol{x} = A\boldsymbol{y}$.
- Solution set $\mathcal{Y}(\boldsymbol{x})$ can be huge.
- Enumeration computationally infeasible.

Sampling Based Methods

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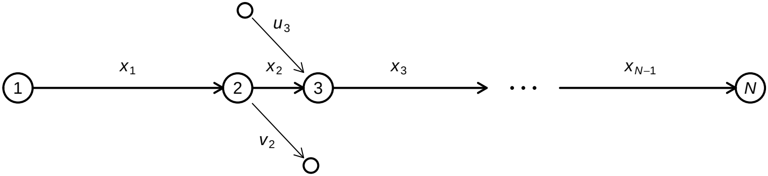
- Problem – typically can't enumerate set $\mathcal{Y}(\boldsymbol{x})$.
- So how to generate candidates from $\mathcal{Y}(\boldsymbol{x})$ (as part of MCMC)?
- Some solutions proposed, but not entirely reliable.

  e.g. Tebaldi, C. and M. West (1998). Bayesian inference on network traffic using link count data (with discussion). *JASA* **93**, 557–576.

Our Solution for Linear Networks (Highway)



- Build proposal distribution on crude behavioural model.
- Assume drivers forget origin, so all currently on road are equally likely to leave at next exit.
- Easy to simulate conditional on link counts.
- Get proposal probabilities from simple combinatorics.
- Always generates feasible candidate $\boldsymbol{y}$.
- Acceptance probabilities easy to compute.

Likelihood Functions for Day-to-Day Dynamic Models

- Assume travel behaviour on day t depends on costs over finite past (Markov).
- Assume costs depend on link flows (not route).
- Then likelihood factorizes; e.g. for one-day memory

$$L(\boldsymbol{\psi}) = \prod_t \sum_{\boldsymbol{y}^t \in \mathcal{Y}^t(\boldsymbol{x}^t)} f(\boldsymbol{y}^t | \boldsymbol{x}^{t-1}, \boldsymbol{\psi}).$$

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- Hence sampling of route flows can be done day by day.

Closing Comments

- Inference problems for network-based models have complex linear inverse structure.
- Simulation of latent route flows is very challenging.
- Need to sample candidates from huge set of feasible flows.
- We have solution for linear networks.
- Easy to extend to tree networks etc.
- Method for 'unordered' networks remains a major stumbling block.

Acknowledgement

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Reading

- Copy of these slides
  `http://www.massey.ac.nz/~mhazelto/seminars`
- Hazelton, M.L. (2010). Bayesian inference for network-based modes with a linear inverse structure. *Transportation Research Part B* **44**, 674–685.
- Hazelton, M.L. (2010). Statistical inference for transit system origin-destination matrices. *Technometrics* **52**, 221-230.
- Tebaldi, C. and M. West (1998). Bayesian inference on network traffic using link count data (with discussion). *Journal of the American Statistical Association* **93** 557–576.