TEST FOR TIME SERIES & FORECASTING (161.342)

Answer ALL questions. Each part of each question is worth 2 marks.

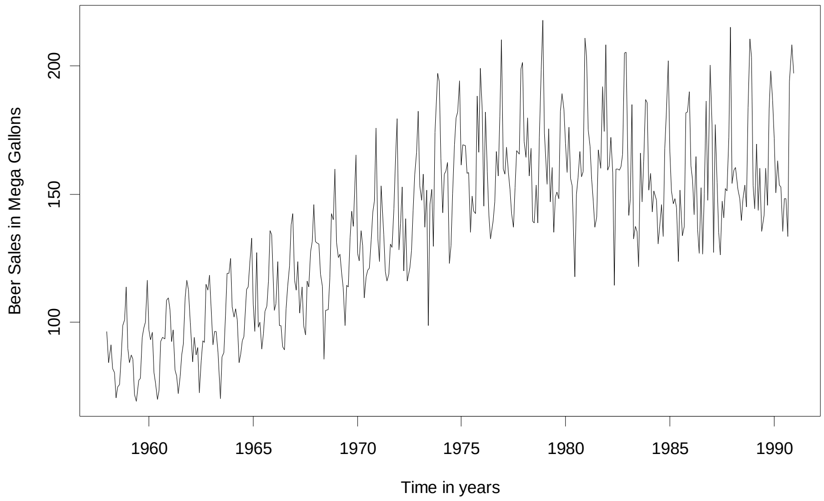
Time allowed = 50 minutes.

Question 1:

Monthly sales of beer from January 1958 to December 1990 in Australia have been read into the time series variable `beer.ts` in the R programming environment.

(a) Comment on the main features evident in the following plot.

(b) Give the R command (with correct parameters) that produced this plot.



A model with trend and seasonal components was fitted. The commands and summary output are shown below.

```
> month <- cycle (beer.ts)
> beer.lm <- lm(beer.ts ~ t + t2 + factor(month))
> summary(beer.lm)


Coefficients:
                   Estimate Std. Error t value Pr(>|t|)
(Intercept)       7.597e+01  2.619e+00  29.010  < 2e-16 ***
t                 5.658e-01  2.058e-02  27.494  < 2e-16 ***
t2               -8.772e-04  5.020e-05 -17.475  < 2e-16 ***
factor(month)2  -9.105e+00  2.874e+00  -3.168 0.001660 **
factor(month)3   1.958e+00  2.874e+00   0.681 0.496109
factor(month)4  -1.193e+01  2.874e+00  -4.149 4.12e-05 ***
factor(month)5  -1.612e+01  2.874e+00  -5.609 3.91e-08 ***
factor(month)6  -3.006e+01  2.874e+00 -10.457  < 2e-16 ***
factor(month)7  -2.067e+01  2.874e+00  -7.191 3.41e-12 ***
factor(month)8  -1.409e+01  2.875e+00  -4.903 1.40e-06 ***
factor(month)9  -1.076e+01  2.875e+00  -3.742 0.000211 ***
factor(month)10  8.144e+00  2.875e+00   2.833 0.004853 **
factor(month)11  1.669e+01  2.875e+00   5.805 1.35e-08 ***
factor(month)12  2.983e+01  2.875e+00  10.377  < 2e-16 ***
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Residual standard error: 11.68 on 382 degrees of freedom
Multiple R-Squared: 0.8831,     Adjusted R-squared: 0.8791
F-statistic:   222 on 13 and 382 DF,  p-value:     0
```

(b) Which three months have the highest beer sales? Explain with reference to the above output.

(c) Forecast beer sales for January 1991.

A model with trend and harmonic components was also fitted to the data. The commands and summary output are given below:

```
> s1 <- sin(2*pi*t/12)
> c1 <- cos(2*pi*t/12)
> s2 <- sin(4*pi*t/12)
> c2 <- cos(4*pi*t/12)
> s3 <- sin(6*pi*t/12)
> c3 <- cos(6*pi*t/12)
> beer.harm.lm <- lm(beer.ts ~ t + t2 + s1 + c1 + s2 + c2 + s3 + c3)
> summary(beer.harm.lm)

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  7.125e+01  1.873e+00  38.034  < 2e-16 ***
t            5.660e-01  2.179e-02  25.979  < 2e-16 ***
t2          -8.772e-04  5.315e-05 -16.505  < 2e-16 ***
s1          -1.069e+00  8.787e-01  -1.216   0.2247
c1           1.979e+01  8.785e-01  22.525  < 2e-16 ***
s2          -5.868e+00  8.785e-01  -6.679 8.40e-11 ***
c2           2.001e+00  8.785e-01   2.278   0.0233 *
s3          -4.142e+00  8.785e-01  -4.715 3.38e-06 ***
c3           5.805e+00  8.785e-01   6.608 1.30e-10 ***
---
Signif. codes:  0 `***' 0.001 `**' 0.01 `*' 0.05 `.' 0.1 ` ' 1

Residual standard error: 12.36 on 387 degrees of freedom
Multiple R-Squared: 0.8673,     Adjusted R-squared: 0.8645
F-statistic: 316.1 on 8 and 387 DF,  p-value:     0
```

(f) Forecast beer sales for January 1991 using the above fitted harmonic model.

(d) Which of the two forecasts (part (c) or (f)) do you think is the most reliable? Give a reason.

(e) Calculate the phase shift of the first harmonic in the above fitted model. At what time of year is the maximum?

Question 2:

(a) Define the AR(1) model in terms of a polynomial in B, where B is the backward shift operator.

(b) Define the MA(1) model in terms of a polynomial in B, where B is the backward shift operator.

(c) Using the polynomial in B, find the range of parameter values for which the AR(1) model is stationary.

(d) Using the polynomial in B, find the range of parameter values for which the MA(1) model is invertible.

(e) Is the following model stationary: $X_t = X_{t-1} - ½ X_{t-2} + a_t - 3a_{t-1}$?