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Gas Holdup in Pneumatic Reactors

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ABSTRACT

The gas holdup expressions of the form $\epsilon_G = aU_{sg}^b$, *often observed to be applicable in bubble columns and airlift reactors, are shown, for the first time, to have a basis in fundamental theory. The mechanistic nature of parameters a and b is explained.*

1. INTRODUCTION

Gas–liquid and gas–liquid–solid reactors are used in many industrial aerobic fermentations, catalytic hydrogenations, hydrodesulfurization and other gas absorption operations. The volume fraction of gas in the gas–liquid dispersion, or the gas holdup, has a strong influence on the performance of these reactors; the residence time of the gas in the liquid, the gas–liquid contact area for mass transfer and the design volume of the reactor depend on the gas holdup which occurs under given operating conditions. In pneumatic reactors such as bubble columns the principal operating influence on gas holdup is the gas velocity in the vessel. Consequently the effect of gas velocity on holdup has been much investigated. Gas holdup dependence on superficial gas velocity has commonly [1 - 9] been expressed in the form

$$\epsilon_G = aU_{sg}^b \tag{1}$$

where b is usually between 0.5 and 1. Another less frequently reported holdup equation is [10, 11]

$$\epsilon_G = \frac{U_{sg}}{\alpha + \beta U_{sg}} \tag{2}$$

for which we found α and β to be 2.70 and 0.284 m s^{-1} respectively, based on extensive data [1, 5, 12, 13] obtained for air–water in bubble columns of diameters between 0.15 and 1.0 m over a superficial gas velocity range of 0.01 - 0.45 m s^{-1}. Equation (2) finds some basis in the drift-flux model [14, 15]. However, eqn. (1) seems to be more widely applicable and it often provides a better fit to the experimental data. In addition, the exponent b (eqn. (1)) shows a sharp change at the point of transition from bubble to coalesced bubble flow, thereby providing a good indication of the reactor flow regime. In this communication eqn. (1) is shown to have a theoretical basis which was not previously understood.

2. THEORY

In a bubble column the gas holdup may theoretically be shown to satisfy the equation

$$\epsilon_G = \frac{U_{sg}}{U_T} \tag{3}$$

where U_T is the mean terminal rise velocity of the gas bubbles. Equation (3) arises from the definition of gas holdup:

$$\epsilon_G = \frac{A_G}{A} \tag{4}$$

where A_G and A are the actual cross-sectional area for the gas flow and the total cross-section of the gas–liquid flow channel respectively; and from the continuity equation

$$A_G U_T = A U_{sg} \tag{5}$$

The actual or terminal rise velocity of the gas bubbles is found by equating the buoyancy and drag forces. The buoyancy force F_B on a bubble is

$$F_B = (\rho_L - \rho_G)\frac{\pi}{6} d_B^3 g \tag{6}$$

where d_B is the Sauter mean bubble diameter. The drag force F_D is

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$$F_D = \frac{C_D U_T^2 \rho_L A_p}{2} \quad (7)$$

assuming the bubble to be a rigid particle, as a first approximation. A_p and C_D are the projected area of the bubble and a dimensionless drag coefficient respectively. Substitution of

$$A_p = \frac{\pi d_B^2}{4} \quad (8)$$

into eqn. (7) gives

$$F_D = \frac{\pi C_D U_T^2 \rho_L d_B^2}{8} \quad (9)$$

Equating eqns. (6) and (9) followed by rearrangement leads to

$$U_T^2 = \frac{4(\rho_L - \rho_G) g d_B}{3 \rho_L C_D} \quad (10)$$

The general form of the drag coefficient dependence on the particle Reynolds number has been given [16] as

$$C_D = \frac{i}{\mathrm{Re}_p^{\,j}} \quad (11)$$

where i and j depend on the flow regime, and the particle Reynolds number Re_p is

$$\mathrm{Re}_p = \frac{(\rho_L - \rho_G) U_T d_B}{\mu_L} \quad (12)$$

Substitution of eqn. (12) in eqn. (11) and of the resulting equation in eqn. (10) yields the expression

$$U_T = \left\{ \frac{4(\rho_L - \rho_G)^{1+j} g d_B^{\,1+j}}{3 \rho_L i \mu_L^{\,j}} \right\}^{1/(2-j)} \quad (13)$$

Because for a given gas–liquid system the fluid properties are almost always constant, eqn. (13) may be rewritten as

$$U_T = C d_B^{\,x} \quad (14)$$

where C and x are functions only of the two-phase flow regime. The bubble size in eqn. (14) may be predicted from the well-known Kolmogoroff's theory of local isotropic turbulence because most pneumatic reactors, with the possible exception of those using highly viscous media, are found to satisfy the isotropic turbulence criteria. Hence, the bubble diameter may be expressed in the form [17]

$$d_B = \phi \frac{\sigma^{0.6}}{(P_G/V_L)^{0.4} \rho_L^{\,0.2}} \quad (15)$$

where ϕ is a constant and P_G/V_L is the power input per unit liquid volume in the reactor. The power input into a pneumatic device is predominantly due to isothermal gas expansion and it is related analytically to the superficial gas velocity in the following manner:

$$\frac{P_G}{V_L} = \rho_L g U_{sg} \quad (16)$$

the superficial gas velocity being obtained from

$$U_{sg} = \frac{Q_m R T}{A L \rho_L g} \ln\left(1 + \frac{\rho_L L g}{P_h}\right) \quad (17)$$

where Q_m is the molal gas flow. Substitution of eqns. (15) and (16) into eqn. (14) gives

$$U_T = C \left(\frac{\phi \sigma^{0.6}}{\rho_L^{\,0.2}} \right)^x (\rho_L g U_{sg})^{-0.4x} \quad (18)$$

or

$$U_T = \psi U_{sg}^{\,y} \quad (19)$$

which when substituted in eqn. (3) results in an equation of the same form as eqn. (1), thereby establishing its basis in fundamental theory. The coefficient a in eqn. (1) has the physical meaning

$$a = \left[\frac{4g}{3 \rho_L \mu_L^{\,j} i} \left\{ \frac{(\rho_L - \rho_G) \sigma^{0.6} \phi}{\rho_L^{\,0.2}} \right\}^{1+j} \right]^{1/(j-2)} \quad (20)$$

where i and j must be empirically determined. Dependence of a on fluid properties and on the flow regime is indicated.

3. RESULTS AND DISCUSSION

In Fig. 1 we have plotted the gas holdup data from several sources, including our data, for air–water in bubble columns according to eqn. (1). A very good fit of the data is shown in Fig. 1, the a and b values being 2.47 and 0.97, and 0.49 and 0.46, respectively, for the bubble ($U_{sg} < 0.05$ m s^{-1}) and coalesced-bubble flow ($U_{sg} > 0.05$ m s^{-1}) regimes.

The break in the data (Fig. 1) at a superficial gas velocity of 0.05 m s^{-1} marked the transition from bubble to coalesced bubble flow. In the bubble flow regime the gas bubbles were spheroidal and bubble–bubble interaction was small. The turbulence-determined bubble size fell between 0.001 and 0.01 m and the terminal bubble-rise velocity was nearly constant owing to the alternate

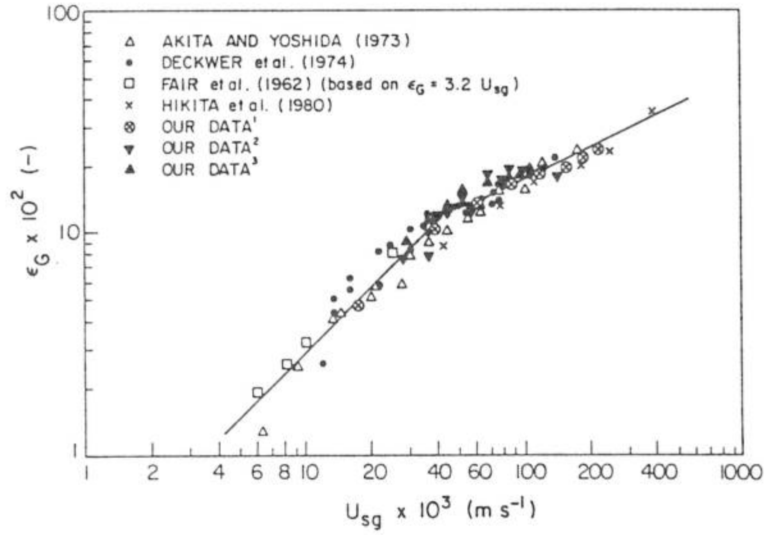

Fig. 1. Gas holdup *vs.* superficial air velocity in bubble columns (air–water); the data shown cover column diameter and height ranges, respectively, of 0.10 - 1.067 m and 1.37 - 5.87 m; superscripts: 1, rectangular bubble column: liquid height, 1.37 m; cross-section, 0.46 × 0.155 m; 2, circular bubble column; liquid height, 5.87 m; diameter, 0.243 m; 3, liquid height, 1.50 m; diameter, 0.243 m.

vortex shedding behind spheroidal bubbles. Gas holdup was then almost directly proportional to superficial gas velocity as confirmed experimentally.

For superficial gas velocities in excess of 0.05 m s^{-1} gas holdup became sufficiently high that bubble–bubble interactions led to coalescence and spherical cap bubbles were formed. Under those conditions eqn. (15) was not entirely satisfactory because it did not consider bubble–bubble interactions. Spherical cap bubbles rose faster than spheroidal bubbles and hence the gas holdup dependence on superficial gas velocity declined sharply.

Clearly, because i and j are dependent on the shape of the body past which a fluid flows, these parameters changed with bubble shape.

For the bubble flow regime, i and j were calculated to be 1.8×10^{-5} and -1.243 respectively, in air–water. Hence the drag coefficient for a gas bubble in this regime was

$$C_D = 1.8 \times 10^{-5}\ Re_p^{1.243} \qquad (21)$$

Equation (21) produced quite reasonable values for the drag coefficient of bubbles (approximately 4 - 8 mm in diameter) when compared with solid spheres. Additionally, the dimensional consistency constraint on eqn. (1) meant that a had the units $(m\ s^{-1})^{-b}$. These when equated to the expression for a given in eqn. (20) led to $j = -1.0$, which was quite close to the value of -1.243 calculated on the basis of experimental observations shown in Fig. 1. A theoretical value for b of 1.0 was predicted and this was in good agreement with the experimental value of 0.97.

4. CONCLUSIONS

Gas holdup dependence on superficial gas velocity is satisfactorily described by equations of the form

$$\epsilon_G = aU_{sg}{}^b$$

which have a definite hydrodynamic basis. The parameters a and b depend on the two-phase flow regime in the reactor.

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APPENDIX A: NOMENCLATURE

A	cross-sectional area of reactor (m^2)
A_G	cross-sectional area occupied by gas (m^2)
A_p	projected area of bubble (m^2)
a	coefficient in eqn. (1) $(m\ s^{-1})^{-b}$
b	exponent in eqn. (1)
C	constant in eqn. (14) ($m^{1-x}\ s^{-1}$)
C_D	drag coefficient
d_B	Sauter mean bubble diameter (m)
F_B	buoyancy force (kg m s^{-2})
F_D	drag force (kg m s^{-2})
g	gravitational acceleration (m s^{-2})
i	parameter in eqn. (11)
j	parameter in eqn. (11)
L	unaerated liquid height (m)
P_G	power input due to gassing (W)
P_h	reactor headspace pressure (N m^{-2})
Q_m	molar gas flow (kmol s^{-1})
R	gas constant (kJ K^{-1} $kmol^{-1}$)
Re_p	bubble Reynolds number defined by eqn. (12)
T	absolute temperature (K)
U_{sg}	superficial gas velocity defined by eqn. (17) (m s^{-1})
U_T	true gas velocity or terminal rise velocity (m s^{-1})
V_L	unaerated liquid volume (m^3)
x	exponent in eqn. (14)
y	exponent in eqn. (19)

Greek symbols

α	parameter in eqn. (2) (m s^{-1})
β	parameter in eqn. (2)
ϵ_G	fractional gas holdup
μ_L	viscosity of liquid (Pa s)
π	pi
ρ_G	density of gas (kg m^{-3})
ρ_L	density of liquid (kg m^{-3})
σ	interfacial tension (N m^{-1})
ϕ	parameter in eqn. (15)
ψ	parameter in eqn. (19) $(m\ s^{-1})^{1-y}$