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Prediction of Liquid Circulation Velocity in Airlift Reactors with Biological Media

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ABSTRACT

A method is developed and demonstrated for the prediction of liquid circulation velocity in airlift bioreactors operating with pseudoplastic fluids, including mould suspensions. The method is based on a combination of a recently developed energy balance over airlift loops with analytical expressions for shear stress in turbulent or laminar flow of non-Newtonian fluids. The procedure enables, an a priori *calculation of pseudoplastic fluid circulation in airlift devices for design purposes.*

Key words: Airlift bioreactors, liquid circulation, non-Newtonian flow, pressure drop.

NOTATION

A	Cross-sectional area (m^2)
C	Parameter defined by eqn (16)
d	Equivalent hydraulic diameter (m)
E_B	Energy loss in bottom of reactor (W)
E_D	Energy dissipation in downcomer (W)
E_F	Energy loss due to friction (W)
E_i	Energy input to reactor (W)
E_R	Energy dissipation in riser (W)
E_T	Energy loss in headspace of reactor (W)
g	Gravitational acceleration (ms^{-2})
h	Height (m)
K	Consistency index (Pas^n)

K_B	Frictional loss coefficient for the bottom of reactor
K_T	Frictional loss coefficient for the top of reactor
$k_L a_L$	Overall volumetric gas–liquid mass transfer coefficient (s^{-1})
L	Length of riser or downcomer (m)
n	Flow behaviour index
ΔP_F	Frictional pressure drop (Pa)
Re	Reynolds number defined by eqn (15)
U	Superficial velocity (ms^{-1})
U_H	Heat transfer coefficient ($Wm^{-2}\,°C^{-1}$)
w	Concentration of mycelial solids (dry) ($kg\ m^{-3}$)
x	parameter defined by eqn (17)
ε	Fractional gas holdup
γ	Shear rate (s^{-1})
μ	Apparent viscosity (Pa.s)
ρ	Density ($kg\ m^{-3}$)
τ	Shear stress (Pa)

Subscripts

b, Interconnecting area between riser and downcomer; D, gas–liquid dispersion; d, downcomer; g, gas; L, liquid; r, riser.

INTRODUCTION

Airlift reactors are increasingly being applied to biotechnology-based production processes; numerous examples of successful use of these devices in bacterial, yeast, fungal and tissue culture fermentations have been cited.[1]

Airlift reactors are distinguished by a well defined, gas-induced, circulation of fluid: upflow in the riser and downflow in the downcomer. The velocity of the induced circulation is a major design characteristic[1] of these systems. Important bioreactor performance parameters such as mass and heat transfer, mixing, turbulence and shear levels to which the microorganisms are exposed, are dependent on the circulation of liquid (Fig. 1), sometimes in complex ways. Consequently, the estimation of the induced liquid circulation has recently received some attention[2–5] and the prediction of circulation velocity is now possible, but only for Newtonian and water-like fluids.[1] Many commercially useful fermentations, involving viscous, non-Newtonian fluids have also been successfully carried out in airlift fermenters. For these viscous fluids, however, the prediction of induced circulation is not yet possible and, as a result, the reactor design relies heavily on empiricism.

In the following sections a procedure is developed, which, for the first time, enables an *a priori* estimation of airlift liquid circulation for pseudoplastic, non-Newtonian, media.

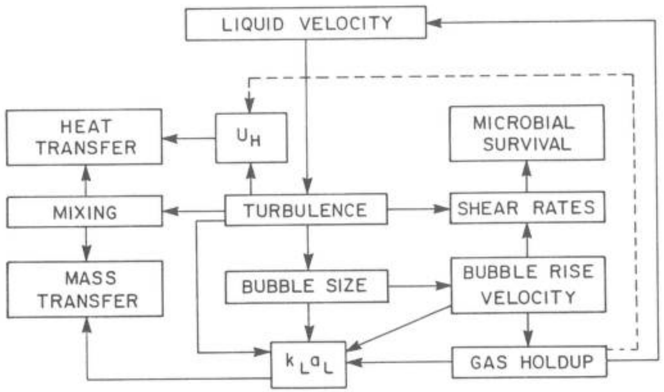

Fig. 1. Interrelationship between the liquid circulation velocity and the other bioreactor performance parameters.

THEORY

As for Newtonian fluids, so also for non-Newtonian media, the energy balance over an airlift loop may be written[2] as:

$$E_i = E_R + E_D + E_B + E_T + E_F \tag{1}$$

where

E_i = energy input to reactor due to isothermal gas expansion;
E_R = energy dissipation due to wakes behind rising bubbles in the riser;
E_D = energy loss due to bubbles entrained in the downcomer against the buoyancy-determined direction of flow;
$E_{B(T)}$ = energy loss due to fluid turn around at the bottom (top) of the reactor;
E_F = energy loss due to friction in the riser and downcomer.

In contrast to water-like systems, the frictional losses (E_F) cannot be ignored when the fluid is viscous.

An earlier development[2] has shown that E_R, E_D, and $(E_B + E_T)$ are given, respectively, by the following equations:

$$E_R = E_i - \rho_L g h_D U_{Lr} A_r \varepsilon_r \tag{2}$$

$$E_D = \rho_L g h_D U_{Ld} A_d \varepsilon_d \tag{3}$$

$$E_B + E_T = \frac{1}{2} \rho_L U_{Lr}^3 A_r \left[\frac{K_T}{(1-\varepsilon_r)^2} + K_B \left(\frac{A_r}{A_d} \right)^2 \frac{1}{(1-\varepsilon_d)^2} \right] \tag{4}$$

The frictional energy dissipation (E_F) is made up of two terms: the dissipation in the riser (E_{Fr}) and that in the downcomer (E_{Fd}), given, respectively, as:

$$E_{Fr} = A_r U_{Lr} \, \Delta P_{Fr} \tag{5}$$

$$E_{Fd} = A_d U_{Ld} \, \Delta P_{Fd} \tag{6}$$

Substitution of eqns (2)–(6) into eqn (1), taking into account the continuity criterion,

$$U_{\mathrm{Lr}}A_{\mathrm{r}}=U_{\mathrm{Ld}}A_{\mathrm{d}} \tag{7}$$

followed by rearrangement, leads to

$$U_{\mathrm{Lr}}A_{\mathrm{r}}(\Delta P_{\mathrm{Fr}}+\Delta P_{\mathrm{Fd}})-\rho_{\mathrm{L}}gh_{\mathrm{D}}U_{\mathrm{Lr}}A_{\mathrm{r}}(\varepsilon_{\mathrm{r}}-\varepsilon_{\mathrm{d}}) + \frac{1}{2}\rho_{\mathrm{L}}U_{\mathrm{Lr}}^{3}A_{\mathrm{r}}\left[\frac{K_{\mathrm{T}}}{(1-\varepsilon_{\mathrm{r}})^{2}}+K_{\mathrm{B}}\left(\frac{A_{\mathrm{r}}}{A_{\mathrm{d}}}\right)^{2}\frac{1}{(1-\varepsilon_{\mathrm{d}})^{2}}\right]=0 \tag{8}$$

For low viscosity Newtonian fluids the frictional pressure losses have been found to be negligible[2] and for such fluids eqn (8) has been shown[2] to be widely applicable for the calculation of U_{Lr}, the parameter of interest. For non-Newtonian media, a similar simplification of eqn (8) is not possible because the frictional pressure drop terms (eqn (8)) can not be ignored.

In what follows we show how eqn (8) may be used for the calculation of U_{Lr} for shear-thinning fluids.

In a manner analogous to flow in pipes,[6] the frictional pressure losses in the riser and the downcomer may be expressed as

$$\Delta P=4\frac{L}{d}\tau_{\mathrm{D}} \tag{9}$$

The velocity prediction problem is then reduced to one of calculation of wall shear stress (τ_{D}) in gas–liquid systems when the liquid is shear-thinning. Two possible flow situations—laminar or turbulent flow—may exist and these will be considered separately.

Laminar flow

For non-Newtonian pseudoplastic media the flow is laminar[7] when

$$\mathrm{Re}\leqslant 2300\,\frac{3n+1}{4n} \tag{10}$$

For this condition the shear stress for the flow of gas-free liquid[8] is

$$\tau_{\mathrm{L}}=\frac{8\rho_{\mathrm{L}}U_{\mathrm{L}}^{2}}{\mathrm{Re}} \tag{11}$$

and for the flow of gas–liquid dispersion this stress is given[8] by

$$\frac{\tau_{\mathrm{D}}}{\tau_{\mathrm{L}}}=\left(\frac{1}{1-\varepsilon}\right)^{n}\left[1+\frac{C(1-\varepsilon)^{1+n}}{\left(\dfrac{\tau_{\mathrm{L}}}{\rho_{\mathrm{L}}}\right)^{(1+n)/n}\left[1+\dfrac{C(1-\varepsilon)^{1+n}}{(\tau_{\mathrm{L}}/\rho_{\mathrm{L}})^{(1+n)/n}}\right]^{n/2}}\right]^{n^{2}/[2(1+n)]} \tag{12}$$

Turbulent flow

Turbulent flow exists for Reynolds numbers higher than those given by eqn (10). For this regime we have[7,8]

$$\tau_{\mathrm{L}}=\frac{0{\cdot}316n^{0{\cdot}121}\rho_{\mathrm{L}}U_{\mathrm{L}}^{2}}{8\,\mathrm{Re}^{2/[n(x-1)+2]}} \tag{13}$$

and

$$\frac{\tau_D}{\tau_L}=\left(\frac{1}{1-\varepsilon}\right)^{2xn/[n(x-1)+2]} \times \left\{1+\frac{C(1-\varepsilon)^{2x(1+n)/[n(x-1)+2]}}{\left(\frac{\tau_L}{\rho_L}\right)^{(1+n)/n}\left[1+\frac{C(1-\varepsilon)^{2x(1+n)/[n(x-1)+2]}}{(\tau_L/\rho_L)^{(1+n)/n}}\right]^{xn/[n(x-1)+2]}}\right\}^{xn^2/[(n+1)[n(x-1)+2]]} \quad (14)$$

Throughout, the Reynolds number and the parameters C and x are given,[7] respectively, by:

$$\mathrm{Re}=\frac{8U_L^{2-n}d^n\rho_L}{K\left(\frac{6n+2}{n}\right)^n} \quad (15)$$

$$C=4^{(1+n)/n}\left(\frac{K}{\rho_L}\right)^{1/n} g\varepsilon(1-\varepsilon)^2\left(\frac{U_g}{\varepsilon}-\frac{U_L}{1-\varepsilon}\right) \quad (16)$$

$$x=0{\cdot}70n^{-0{\cdot}591} \quad (17)$$

Equation (17) was obtained using the data reported by Metkin and Sokolov.[7]

Equations (11) to (14) were developed theoretically and were successfully used by Metkin and Sokolov[8] for the calculation of τ_D/τ_L in the flow of *Aspergillus niger* suspensions ($w=5{\cdot}25$ to $19{\cdot}8$ kg m^{-3}; $n=0{\cdot}787$ to $0{\cdot}92$; $K/\rho_L=5{\cdot}14\times10^{-4}$ to $8{\cdot}85\times10^{-5}$ m^2 s^{n-2}) in vertical pipes ($d=0{\cdot}05$ and $0{\cdot}5$ m) over a broad range of air (0·05 to 0·4 m s^{-1}) and slurry (0·13 to 1·2 m s^{-1}) velocities. The experimental and the predicted[8] values of the ratio τ_D/τ_L were within $\pm 12\%$.

The foregoing equations will be used in eqn (8) for liquid velocity predictions employing the procedure given below.

Prediction of circulation velocity, U_{Lr}

1. Initially, a value of U_{Lr} is assumed. Either ε_r must be known or it must be estimated using an equation[8] such as

$$\varepsilon=\frac{U_g}{U_g+U_L+0{\cdot}9(1+20w\sqrt{\rho_L})} \quad (18)$$

   or other similar correlations reported, for example by Chisti *et al.*[1] Equation (18) is for *A. niger* broths in airlift reactors.
2. Re_r and Re_d are calculated (eqns (15) and (7)), followed by x (eqn (17)). Depending on whether the flow is laminar or turbulent, either eqn (11) or (13) is used to calculate τ_{Ld} and τ_{Lr}. C_r is then calculated (eqn (16)).
3. The shear stress τ_{Dr} (eqns (12) or (14)) is calculated. Note that for many cases, particularly for the external loop type of airlift reactors, the downcomer gas holdup (ε_d) is close to zero and the calculation of τ_{Dd} is not necessary. When ε_d

is not zero, equations such as those reported by Chisti *et al.*[1] may be used to calculate it; τ_{Dd} is then calculated (eqns (12) or (14)).

4. The pressure drops ΔP_{Fr} and ΔP_{Fd}, are calculated using eqn (9).
5. The dispersion height is obtained as follows:

$$h_{\mathrm{D}}=\frac{h_{\mathrm{L}}}{1-\left(\dfrac{\varepsilon_{\mathrm{r}}A_{\mathrm{r}}+\varepsilon_{\mathrm{d}}A_{\mathrm{d}}}{A_{\mathrm{r}}+A_{\mathrm{d}}}\right)} \tag{19}$$

6. The frictional loss coefficients for the headspace (K_{T}) and the bottom (K_{B}) of the reactor are calculated: For internal loop airlifts[2]

$$K_{\mathrm{B}}=11\cdot402\left(\frac{A_{\mathrm{d}}}{A_{\mathrm{b}}}\right)^{0\cdot789} \tag{20}$$

while K_{T} is negligible;[2] and for external loop devices[2] $K_{\mathrm{T}}\approx K_{\mathrm{B}}\approx 5$.

7. Equation (8) is now solved for U_{Lr} using the already calculated values of ΔP_{Fr}, ΔP_{Fd}, h_{D}, ε_{r}, ε_{d}, K_{T} and K_{B}. Either a manual trial and error method or a computer software package for evaluating the zeros of a function must be employed.
8. The U_{Lr} calculated in step 7 above is compared with the value that was initially assumed (step 1). If these two values are comparable then the problem is solved. Otherwise the foregoing procedure must be repeated with a new assumption for U_{Lr}.

The only experimental measurements of liquid circulation in airlift reactors with pseudoplastic media are those of Popovic and Robinson.[9] Here we compare the U_{Lr} predicted using our method, for their system with the experimental liquid circulation velocity reported by those investigators.[9] Popovic and Robinson[9] employed aqueous carboxymethyl cellulose solutions covering an apparent viscosity range of about 0·015 to 0·5 Pas. They used external loop type of airlifts ($d_{\mathrm{r}}=0\cdot152$ m throughout; $A_{\mathrm{d}}/A_{\mathrm{r}}=0\cdot111$, 0·25 and 0·444; $h_{\mathrm{D}}\approx 1\cdot8$ m) with a maximum liquid volume of about 50 dm^3. The experimental gas holdup and liquid velocity were correlated,[9] respectively, by the equations

$$\varepsilon_{\mathrm{r}}=0\cdot02U_{\mathrm{gr}}^{0\cdot6504}\left(1+\frac{A_{\mathrm{d}}}{A_{\mathrm{r}}}\right)^{-1\cdot0516}\mu^{-0\cdot1039} \tag{21}$$

and

$$U_{\mathrm{Lr}}=0\cdot052U_{\mathrm{gr}}^{0\cdot322}\left(\frac{A_{\mathrm{d}}}{A_{\mathrm{r}}}\right)^{0\cdot794}\mu^{-0\cdot395} \tag{22}$$

The apparent viscosity in eqns (21) and (22) was defined[9] as

$$\mu=K\gamma^{n-1} \tag{23}$$

with the shear rate (γ) calculated using

$$\gamma=5000U_{\mathrm{gr}} \tag{24}$$

The gas velocity in eqns (21) and (22) is in cm s^{-1}, all other terms being in SI units.

TABLE 1
Gas Velocity, Fluid Properties, and the Reynolds Numbers used in Calculations

U_{gr} ($m\ s^{-1}$)	K (Pas^n)	n	ε_r	Ad/Ar	d_r (m)	d_d (m)	Re_r	Re_d	U_{Lr} experimental ($m\,s^{-1}$)
0·02	0·38	0·6	0·0286	0·444	0·152	0·102	89	219	0·1035
0·06	0·38	0·6	0·0611	0·444	0·152	0·102	185	454	0·1754
0·12	0·38	0·6	0·0987	0·444	0·152	0·102	276	677	0·2446
0·02	1·6	0·59	0·0247	0·444	0·152	0·102	2	6	0·0597
0·06	1·6	0·59	0·0529	0·444	0·152	0·102	13	32	0·1017
0·12	1·6	0·59	0·0856	0·444	0·152	0·102	28	70	0·1422
0·02	0·29	0·7	0·0280	0·444	0·152	0·102	97	212	0·0960
0·12	0·29	0·7	0·095	0·444	0·152	0·102	272	590	0·2114
0·18	0·29	0·7	0·1252	0·444	0·152	0·102	333	723	0·2527
0·02	1·3	0·64	0·0247	0·444	0·152	0·102	5	11	0·0592
0·12	1·3	0·64	0·0846	0·444	0·152	0·102	33	77	0·1360
0·02	0·25	0·7	0·0375	0·111	0·152	0·051	18	144	0·0339
0·06	0·25	0·7	0·0792	0·111	0·152	0·051	40	323	0·0549
0·12	0·25	0·7	0·1271	0·111	0·152	0·051	62	502	0·0746
0·02	0·25	0·73	0·0326	0·25	0·152	0·076	57	199	0·0611
0·12	0·25	0·73	0·1100	0·25	0·152	0·076	163	571	0·1317
0·18	0·25	0·73	0·1449	0·25	0·152	0·076	201	703	0·1567

For several values of gas velocity (Table 1), covering a range which is typical for such reactors, we calculated ε_r and U_{Lr} using eqns (21) and (22). This U_{Lr} was compared with that calculated using eqns (8), (9), (11), (12), (15) and (16) as outlined earlier. As shown in Table 1, the flow regime in the riser and the downcomer was always laminar. The K_B and K_T used in eqn (8) were both taken to be 6·09 in keeping with earlier observations[2] for reactors identical to those employed by Popovic and Robinson.[9] The downcomer gas holdup (ε_d) was assumed to be negligible in keeping with well known observations (Chisti and Moo-Young;[1] Verlaan *et al.*[10]) for external loop airlifts. The K and n in Table 1 were determined[9] in coaxial cylinder viscometers.

The theoretically calculated velocity is shown plotted (Fig. 2) against the corresponding value obtained by the experimental[9] equations. As can be seen in Fig. 2, the agreement between theoretical and experimental results is excellent; particularly if it is noted that the prediction using the foregoing procedure is quite *a priori*. This confirms that the recommended procedure may be successfully employed for liquid velocity prediction either for airlift reactor design or for reactor operational purposes.

CONCLUSION

A technique based on the hydrodynamic fundamentals has been developed for *a priori* calculation of liquid circulation velocity in airlift devices operating with non-

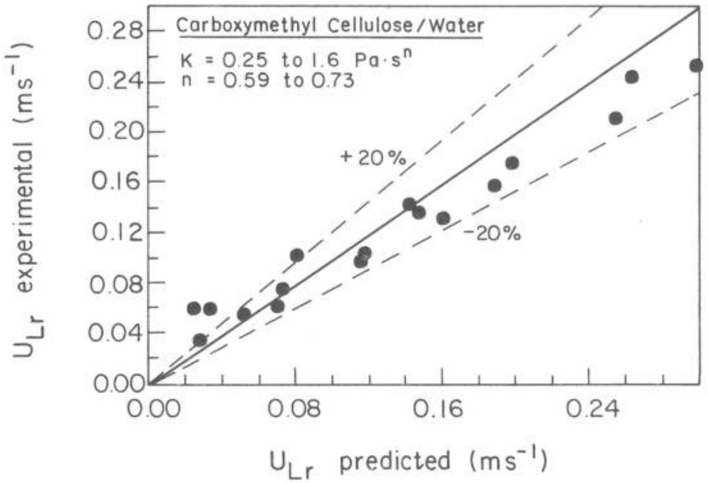

Fig. 2. Predicted versus experimental superficial liquid velocity in the riser.

Newtonian fluids. The technique predicts the liquid velocity very well and will be found useful for airlift bioreactor design.

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