

# Relationship between riser and downcomer gas hold-up in internal-loop airlift reactors without gas–liquid separators

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## Abstract

An approach based on mechanistic principles was used to point out a relationship between the gas hold-ups in the riser and downcomer of internal-loop airlift reactors without gas–liquid separators. The theoretically based relationship was shown to apply to experimental observations in five reactors, including split-cylinder and concentric draft-tube types of reactor configurations, over a broader range of volumes ( $0.033\text{--}0.160\text{ m}^3$ ), heights (1.8–3.45 m) and riser-to-downcomer cross-sectional area ratios (1.8–7.7) than any of the previously published empirical correlations. Unlike earlier equations, the proposed equation was consistent with the observation that, at low rates of gas input in the riser, there may be no gas in the downcomer.

## 1. Introduction and review

Airlift reactors consist of a column of liquid (or slurry) divided into a gas-sparged riser and a downcomer that is usually not aerated. In internal-loop types of airlift devices, the division into riser and downcomer is achieved either by installation of a concentric draft-tube in the cylindrical column of liquid or by a tightly fitting vertical baffle to give a split-cylinder geometry as illustrated in Fig. 1. The riser and the downcomer are interconnected near the top and the bottom of the reactors. These types of reactors are commonly used in gas–liquid contacting in the chemical process industry, biotechnology-based production and environmental waste treatment such as the activated sludge biological treatment of wastewater [1–3].

The volume fraction of gas in the dispersion, or gas hold-up, which exists under given operating conditions is an essential parameter for the design of airlift reactors. The gas-sparged riser has a higher gas hold-up than the downcomer and this difference in hold-up causes liquid circulation in the reactor. The gas and liquid flow upward in the riser and down through the downcomer. While the downcomer may not be sparged with gas, in reactors without gas–liquid separators in the head region as in Fig. 1, a large proportion of the gas bubbles exiting the

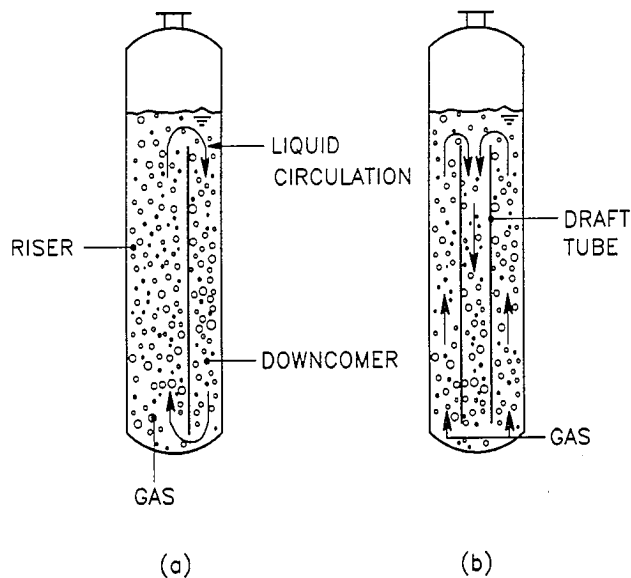


Fig. 1. The (a) split-cylinder and (b) concentric draft-tube geometries of internal-loop airlift reactors.

riser are dragged into and down the downcomer by the recirculating liquid. Many internal-loop airlift devices operate without gas–liquid separators. Although the separators are easy to design [4] and can ensure a downcomer zone which is nearly free of gas at high liquid circulation velocities, they are not always necessary. In reactors without separators, the gas hold-up in the downcomer depends mainly on the hold-up in the riser and the velocity of the circulating liquid. Many aspects of performance of

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airlift reactors depend not only on the overall gas hold-up but also on the distribution of hold-up between the riser and the downcomer; hence, a knowledge of this distribution is essential.

Apart from the rate of induced liquid circulation, some other operating variables which are affected by gas hold-up are the gas-liquid interfacial area for mass transfer, the residence time of gas in the liquid and the residence time of liquid in the riser and the downcomer. The specific gas-liquid interfacial areas  $a_L$  for mass transfer are

$$a_{Lr} = \frac{6\epsilon_r}{d_{Br}(1-\epsilon_r)} \quad (1)$$

and

$$a_{Ld} = \frac{6\epsilon_d}{d_{Bd}(1-\epsilon_d)} \quad (2)$$

in the riser and the downcomer respectively. In these equations,  $\epsilon_r$  and  $\epsilon_d$  are the gas hold-ups in the riser and the downcomer respectively, and  $d_{Br}$  and  $d_{Bd}$  are the mean bubble diameters in the two zones. Because in internal-loop airlift devices the heights of the riser and the downcomer are approximately equal, the ratio of the gas-liquid interfacial areas  $A_{Lr}$  and  $A_{Ld}$  in the riser and the downcomer zones becomes

$$\frac{A_{Lr}}{A_{Ld}} = \frac{A_r}{A_d} \frac{d_{Bd}}{d_{Br}} \frac{\epsilon_r}{\epsilon_d} \quad (3)$$

where  $A_r$  and  $A_d$  are the cross-sectional areas of the riser and the downcomer respectively. Generally, the mean bubble diameter  $d_{Bd}$  in the downcomer is slightly lower than that  $d_{Br}$  in the riser because smaller bubbles are more easily and preferentially dragged into the downcomer [1, pp. 132-149].

Similarly, the residence times  $t_{Ld}$  and  $t_{Lr}$  of the recirculating liquid phase in downcomer and riser depend on gas hold-up as follows:

$$t_{Lr} = \frac{h_r(1-\epsilon_r)}{U_{Lr}} \quad (4)$$

for the riser and

$$t_{Ld} = \frac{h_d(1-\epsilon_d)}{U_{Ld}} \quad (5)$$

for the downcomer. Here  $h_d$  and  $h_r$  are the heights of the downcomer and the riser respectively, and  $U_{Lr}$  and  $U_{Ld}$  are the respective superficial liquid velocities in these zones. Again, because the heights are roughly equal and the volumetric liquid flows in the riser and the downcomer are the same, that is,

$$U_{Lr}A_r = U_{Ld}A_d \quad (6)$$

eqns. (4)-(6) can be combined to give

$$\frac{t_{Lr}}{t_{Ld}} = \frac{A_d}{A_r} \frac{1-\epsilon_r}{1-\epsilon_d} \quad (7)$$

Equation (3) and eqn. (7) show how some important properties of airlift reactors depend on the gas hold-up in the riser and the downcomer.

Because of its significance, the relationship between the gas hold-ups in the riser and the downcomer has received some attention in the published literature. Using air-water, Bello [5] established the empirical relationship

$$\epsilon_d = 0.89\epsilon_r \quad (8)$$

which applied to annulus sparged concentric draft-tube airlift reactors (0.033 m<sup>3</sup> approximate dispersion volume;  $A_d/A_r = 0.13, 0.35$  or  $0.56$ ; 0.152 m outer tube diameter in all cases; 1.8 m gas-liquid dispersion height; 1.55 m height of draft tubes measured from the base of the reactors). In a much larger split-cylinder airlift device (0.23 m<sup>3</sup> working volume; 5.0 m liquid height;  $A_d/A_r = 0.411$ ; 0.243 m column diameter; 0.229 m baffle width; 4.8 m baffle height), Chisti [1, p. 218] found a similar correlation

$$\epsilon_d = 0.997\epsilon_r \quad (9)$$

Equation (9) was determined for air-water and air-salt solution (0.15 M sodium chloride). Other investigations, in external-loop airlift reactors, have revealed similar linear relationships between the gas hold-up in the riser and that in the downcomer in simple air-water systems [1, pp. 184-186, 5] as well as more complex fluids [2, 6]. Thus, there is significant empirical evidence spanning different types of airlift devices, ranges of reactor volumes and varieties of fluids, for linear dependence between the gas hold-ups in the riser and the downcomer. An exception is the work of Miyahara *et al.* [7] which was carried out in draft-tube sparged concentric tube airlift reactors of modest size (0.021 m<sup>3</sup> dispersion volume; 0.148 m outer column diameter;  $A_r/A_d = 0.128-0.808$ ; 1.2 m height of dispersion, 1.0 m height of draft tubes) and several fluids ( $\rho_L = 952-1168$  kg m<sup>-3</sup>;  $\mu_L = 1.0 \times 10^{-3} / -14.9 \times 10^{-3}$  Pa s;  $\sigma = 34.1 \times 10^{-3} / -72.0 \times 10^{-3}$  N m<sup>-1</sup>). Miyahara *et al.* [7] observed the following relationships between gas hold-ups:

$$\epsilon_d = 4.51 \times 10^6 \text{Mo}^{0.115} \left( \frac{A_r}{A_d} \right)^{4.2} \epsilon_r^{4.2} \quad (10)$$

when

$$\epsilon_r < 0.0133 \left( \frac{A_r}{A_d} \right)^{-1.32} \quad (11)$$

and

$$\epsilon_d = 0.05 \text{ Mo}^{-0.22} \left[ \left( \frac{A_r}{A_d} \right)^{0.6} \epsilon_r \right]^{0.31 \text{ Mo}^{-0.073}} \quad (12)$$

when the gas hold-up in the riser was greater than that obtained with the inequality (11). In these equations Mo is the Morton number defined as

$$\text{Mo} = \frac{g \mu_L^4}{\rho_L \sigma^3} \quad (13)$$

Equations (10) and (11) were established in reactors with cross-sectional areas of risers smaller than the areas of downcomers even though the opposite is the norm for practical designs of airlift reactors.

Clearly, the current state of knowledge of the relationship between the riser and the downcomer gas hold-ups in airlift reactors without gas-liquid separators is purely empirical; the fundamental reasons behind the observed behaviour of correlations are not known. In this paper we demonstrate how a previously developed mechanistic relationship for prediction of liquid circulation velocity in airlift devices may be used for predicting gas hold-up in the downcomer of internal-loop airlift reactors. The gas hold-up prediction procedure is experimentally confirmed for split-cylinder and concentric draft-tube types of airlift reactors.

## 2. Theory

Using an energy balance on a liquid circulation loop, Chisti *et al.* [8] developed a theoretical equation for the prediction of liquid circulation velocity in airlift reactors. The equation was shown to apply to low viscosity, water-like, fluids in external- and internal-loop airlift reactors over a broad range of scales. Since initial publication, the equation has been repeatedly validated by other independent investigators [9–13]. The equation, as recommended for internal-loop airlift reactors without gas-liquid separators, is [8]

$$U_{Lr} = \left[ \frac{2gh_D(\epsilon_r - \epsilon_d)}{K_B(A_r/A_d)^2(1 - \epsilon_d)^2} \right]^{0.5} \quad (14)$$

where  $g$  is the gravitational acceleration,  $h_D$  is the height of the gas-liquid dispersion and  $K_B$  is the frictional loss coefficient for the bottom zone of the reactor. For internal-loop reactors, the parameter  $K_B$  is related to the configuration of the bottom

zone as follows [8]:

$$K_B = 11.402 \left( \frac{A_d}{A_b} \right)^{0.789} \quad (15)$$

In eqn. (15),  $A_b$  is the free area for liquid flow under the baffle or the draft tube (Fig. 1).

A simple rearrangement of eqn. (14) leads to

$$\frac{U_{Lr}^2 K_B (A_r/A_d)^2}{2gh_D(\epsilon_r - \epsilon_d)(1 - \epsilon_d)^2} = 1 \quad (16)$$

Equation (16) is an implicit relationship between the gas hold-ups in the riser and the downcomer. When the superficial liquid velocity  $U_{Lr}$  in the riser, the height  $h_D$  of the gas-liquid dispersion and the gas hold-up  $\epsilon_r$  in the riser are known, the equation may be used for an iterative solution for the downcomer gas hold-up  $\epsilon_d$  in a reactor of given geometry (given  $A_r$ ,  $A_d$  and  $A_b$ ). The iterative procedure starts with an assumed value for the downcomer gas hold-up  $\epsilon_d$ . The left-hand side of eqn. (16) is computed and compared with the right-hand side. An agreement between the two sides indicates a satisfactory value of  $\epsilon_d$ , the desired variable.

Unlike such previous empirical correlations as eqn. (8) and eqn. (9), eqn. (16) shows the effects of the geometry of airlift reactors, that is the effects of  $K_B$ ,  $h_D$ ,  $A_r$  and  $A_d$ , on the relationship between the gas hold-ups in the riser and the downcomer. Because the equation accounts for reactor-geometry-associated effects, it has the potential for general applicability to internal-loop reactors irrespective of scale.

## 3. Experimental details

Measurements were done in two split-cylinder airlift reactors as shown in Fig. 2. The reactors consisted of a cylindrical Plexiglas column, 0.243 m in diameter, 8.05 m in overall height, split into a riser and a downcomer by a tightly fitting stainless steel baffle, 0.229 m wide. This arrangement produced a riser-to-downcomer cross-sectional area ratio of 2.44. The equivalent hydraulic diameters of the riser and the downcomer were 0.19 m and 0.102 m respectively. The height  $h_b$  of the baffle was either 1.6 m (reactor 1) or 3.2 m (reactor 2). In both instances the baffles were located 0.102 m above the bottom of the reactors. The static, gas-free, height of liquid was either 1.85 m (reactor 1) or 3.45 m (reactor 2), giving, for both cases, a clearance of 0.148 m between the surface of the liquid and the top of the baffle. The riser was sparged with air at 0.02–0.17 m s<sup>-1</sup> superficial velocity

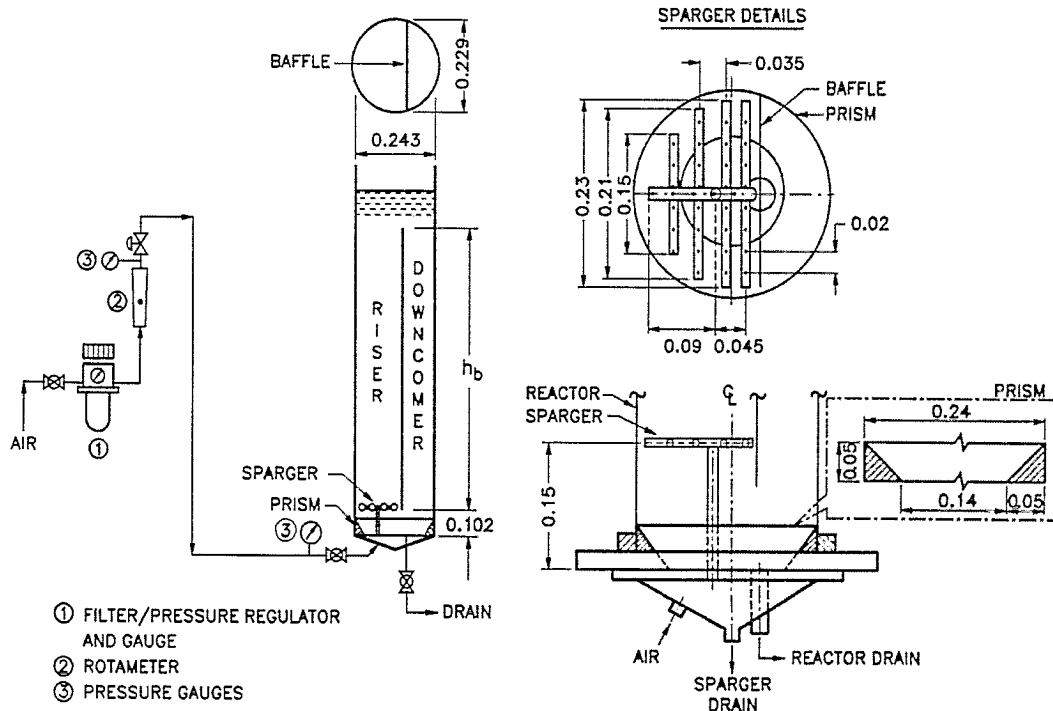


Fig. 2. Details of the split-cylinder reactor and the gas sparger.

based on the cross-sectional area of the riser. A perforated pipe, ladder-type, sparger, as shown in Fig. 2 and previously described by Chisti [1, pp. 94–96], was used for aeration. The sparger had 38 holes,  $1.5 \times 10^{-3}$  m in diameter, giving a free area of 0.2% of the cross-sectional area of the riser. A circular plastic insert placed at the bottom of the reactors (Fig. 2) eliminated stagnant zones and ensured smooth movement of the fluid from the downcomer into the riser. Tap water at 25 °C was the liquid phase in all experiments. Batch operation was employed with respect to the liquid.

Manometric gas hold-up measurements were done in the riser and the downcomer. The vertical distances between the manometric taps in the riser and the downcomer were respectively 1.12 m and 1.08 m in reactor 1 or 2.10 m and 2.06 m in the taller reactor. At any preset gas flow rate, after a hydrodynamic steady state had been attained, the manometer readings were noted and gas hold-ups were calculated with the equation

$$\epsilon = \frac{\rho_L}{\rho_L - \rho_G} \frac{\Delta h}{z} \quad (17)$$

where  $\Delta h$  was the reading on the inverted U-tube manometer and  $z$  was the distance between the pressure taps. Equation (17) is generally applicable and has been commonly used to determine gas hold-up in gas–liquid two-phase systems [14, 15] or in three-phase dispersions when the solid–liquid slurry

can be treated as a pseudo-homogeneous single phase [1, pp. 114–115]. The derivation of eqn. (17) has been detailed elsewhere [1, pp. 294–297]. The overall gas hold-up in the reactor was determined from the increase in level of the two-phase dispersion on introduction of the gas into the system; thus

$$\epsilon = \frac{h_D - h_L}{h_D} \quad (18)$$

where  $h_D$  was the height of the two-phase dispersion and  $h_L$  was the level of gas-free liquid in the reactor.

The velocity of circulating liquid was measured in the downcomer by an acid tracer technique. The acid was injected in the riser and the pH response was noted at two downstream locations in the downcomer in carbonate-free water. The liquid velocity was obtained from the time difference between the responses of the electrodes and the vertical distance between the two probes.

In addition to the two split-cylinder reactors, previously reported data [5] for three-annulus sparged draft-tube airlift devices were used to test further the procedure for predicting the downcomer gas hold-up. The geometric details of these reactors are given in Table 1.

#### 4. Results and discussion

Measured values of the liquid circulation velocity  $U_{Lr}$ , height  $h_D$  of gas–liquid dispersion and the gas

TABLE 1. Geometric details of the draft-tube airlift reactors used to test eqn. (16)

Reactor type	Annulus-sparged concentric tube airlifts
Outer tube diameter (m)	0.152
Downcomer-to-riser cross-sectional area ratio	0.13, 0.35 or 0.56
Height of dispersion (m)	1.80
Height of draft tubes from base of reactor (m)	1.55
System	Air-water

hold-up  $\epsilon_r$  in the riser were used in eqn. (16) to calculate the gas hold-up  $\epsilon_d$  in the downcomer. The calculated hold-ups were compared with the measured or published experimental values for five internal-loop airlift reactors. The data are shown in Fig. 3 and Fig. 4 for respectively the split-cylinder and the draft-tube types of airlift devices. Remarkably good agreement, within  $\pm 30\%$ , is seen between the measured and the calculated gas hold-ups considering that eqn. (16) is purely mechanistic and does not use any empirical parameters. The  $K_B$

values used in the calculations were obtained from eqn. (15). The  $K_B$  value used for the split-cylinder devices was 21.96 which accounted for the flow restrictions associated with the gas sparger [1, pp. 216–217]. However, uncertainty in eqn. (15) was partly responsible for lack of a better than  $\pm 30\%$  agreement between the measured and the calculated hold-ups in Fig. 3. This was confirmed by the results shown in Fig. 5 which is a replot of Fig. 3 with the modification that the  $K_B$  value was used as an adjustable parameter. A best-fit  $K_B$  value of 8.0 improved the agreement between the measured and the calculated downcomer gas hold-ups to better than  $\pm 15\%$  as shown in Fig. 5.

The data in Fig. 3 and Fig. 4 cover a much broader range of reactor volumes ( $0.033\text{--}0.160\text{ m}^3$ ), heights (1.8–3.45 m), riser-to-downcomer cross-sectional area ratios (1.8–7.7) and  $K_B$  values (3.2–21.9) than do previously published equations. Moreover, unlike the empirical eqns. (8)–(10) and eqn. (12) which were developed for specific types of internal-loop airlift systems, eqn. (16) is shown to apply to both main geometric forms, split-cylinder and concentric draft-tube types, of internal-loop airlift reactors. Note that eqn. (14), which is a rearranged form of eqn. (16), has earlier been shown [8] to apply to prediction of the induced liquid circulation velocity

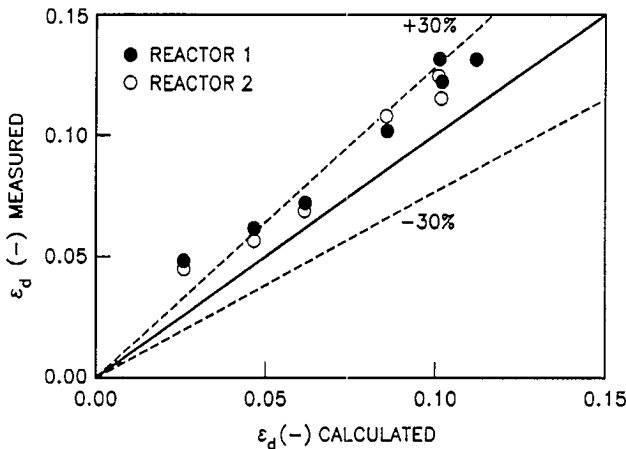


Fig. 3. Parity plot of experimentally measured gas hold-up in the downcomer vs. the calculated values for the two split-cylinder reactors: —, exact agreement.

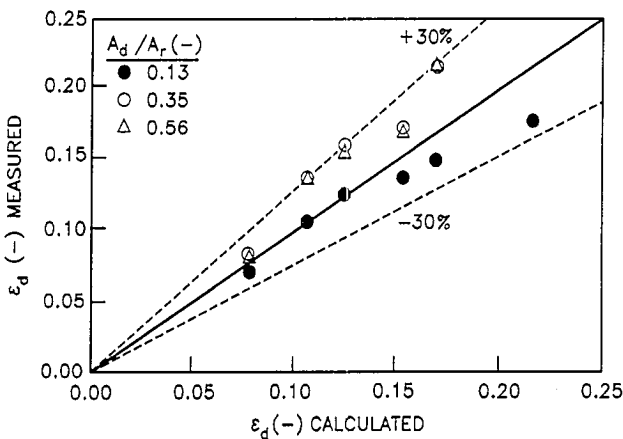


Fig. 4. Parity plot of experimentally observed gas hold-up in the downcomer vs. the calculated values for the three draft-tube airlift reactors of Bello [5]: —, exact agreement.

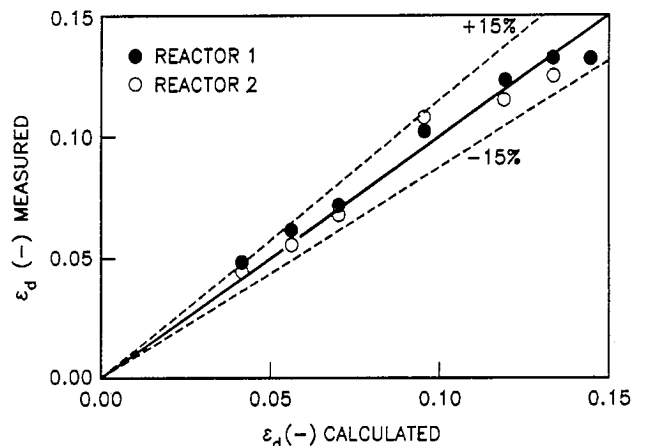


Fig. 5. Parity plot of experimentally measured gas hold-up in the downcomer vs. the calculated values for the two split-cylinder reactors. A best-fit  $K_B$  value of 8.0 was used to obtain the calculated gas hold-ups. —, exact agreement.

in yet larger ranges of geometries of airlift devices than used here to test the equation for prediction of the downcomer gas holdup.

Further, existing empirical correlations, eqns. (8)–(10) and eqn. (12) for example, imply that a gas-free downcomer (*i.e.*  $\epsilon_d = 0$ ) can be achieved only when there is no gas in the riser, that is when  $\epsilon_r = 0$ . This contradicts the well-known observation that at sufficiently low gas injection rates, when the velocity of the induced circulation of liquid is too low to drag any gas into the downcomer, a finite gas hold-up exists in the riser. In this regard, the behaviour of the theoretically developed eqn. (16) is consistent with expectations. Thus, when the downcomer hold-up is nil ( $\epsilon_d = 0$ ), eqn. (16) reduces to the explicit form

$$\epsilon_r = \frac{U_{Lr}^2 K_B (A_r/A_d)^2}{2gh_D} \quad (19)$$

Equation (19) is expected to apply also to internal-loop airlift reactors with gas-liquid separators where, because of gas disengagement in the head region, there is no gas in the downcomer. Of course, for these applications the value of the form frictional loss coefficient  $K_B$  must be revised to account for energy losses in the gas-liquid separator.

## 5. Conclusions

A mechanistic relationship (eqn. (16)) between gas hold-ups in risers and downcomers of internal-loop airlift reactors without gas-liquid separators has been demonstrated for low viscosity, water-like, systems. The relationship shows that gas hold-up in the downcomer depends not only on the hold-up in the riser, as previous empirical correlations have implied, but also on the velocity of induced liquid circulation and the geometric details of the reactor. The theoretical relationship was confirmed by experimental data on split cylinder and concentric draft-tube types of airlift devices over a greater range of scales than any previous correlation. Significantly, the mechanistic eqn. (16) was consistent with the observation that under certain conditions, a gas-free downcomer can be maintained even though the riser may have a non-zero gas hold-up. Previously published correlations, all of which were empirically obtained, could not account for this behaviour which occurs at relatively low values of gas flow rates as would be encountered, for instance, in some animal cell culture applications [16]. Because of its basis in mechanistic principles and demonstrated applicability to a range of sizes and

types of airlift devices, eqn. (16) is expected to have usefulness for scale-up.

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## Appendix A: Nomenclature

- $A_b$  free area for liquid flow under the baffle or draft tube ( $m^2$ )  
 $A_d$  cross-sectional area of downcomer ( $m^2$ )

$A_r$	cross-sectional area of riser ( $\text{m}^2$ )	$h_r$	height of riser (m)
$A_{Ld}$	total gas–liquid interfacial area in downcomer ( $\text{m}^2$ )	$K_B$	frictional loss coefficient for bottom zone of reactor
$A_{Lr}$	total gas–liquid interfacial area in riser ( $\text{m}^2$ )	Mo	Morton number
$a_{Ld}$	specific gas–liquid interfacial area (based on liquid phase) in downcomer ( $\text{m}^{-1}$ )	$t_{Ld}$	liquid residence time in downcomer (s)
$a_{Lr}$	specific gas–liquid interfacial area (based on liquid phase) in riser ( $\text{m}^{-1}$ )	$t_{Lr}$	liquid residence time in riser (s)
$d_{Bd}$	mean bubble diameter in downcomer (m)	$U_{Ld}$	superficial liquid velocity in downcomer ( $\text{m s}^{-1}$ )
$d_{Br}$	mean bubble diameter in riser (m)	$U_{Lr}$	superficial liquid velocity in riser ( $\text{m s}^{-1}$ )
$g$	gravitational acceleration ( $\text{m s}^{-2}$ )	$z$	vertical distance between manometer taps (m)
$\Delta h$	manometer reading (m)	<i>Greek letters</i>	
$h_b$	height of baffle (m)	$\epsilon$	gas hold-up or overall gas hold-up
$h_D$	gas–liquid dispersion height (m)	$\epsilon_d$	gas hold-up in downcomer
$h_d$	height of downcomer (m)	$\epsilon_r$	gas hold-up in riser
$h_L$	gas-free liquid height (m)	$\mu_L$	viscosity of liquid ( $\text{kg m}^{-1} \text{s}^{-1}$ )
		$\sigma$	interfacial tension ( $\text{kg s}^{-2}$ )