

Surname: _____

First name: _____

Student ID: _____

MASSEY UNIVERSITY
Institute of Fundamental Sciences

160.734 (Studies in Applied Differential Equations)

Mid-Semester Test

Semester Two, 2019

Time allowed: **55 minutes**

Total marks: 40

This test is **closed book**. Calculators are permitted.

Attempt all questions. There are 4 questions altogether.

Show all working to receive full credit.

A blank page is provided at the back of the test, in case you need extra space.

1. _____/8

2. _____/10

3. _____/12

4. _____/10

Total: _____/40

1. Let $A = \begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$.

(a) Compute e^{tA} .

(b) Let $\mathbf{x}(t)$ be the solution to $\dot{\mathbf{x}} = A\mathbf{x}$ with $\mathbf{x}(0) = \begin{bmatrix} 1 \\ k \end{bmatrix}$.

For what value of $k \in \mathbb{R}$ does $\mathbf{x}(t)$ converge to the origin as $t \rightarrow \infty$?

[6 + 2 = 8 marks]

2. Consider the system

$$\begin{aligned}\dot{x} &= y + y^2, \\ \dot{y} &= -x.\end{aligned}$$

(a) Find all equilibria.

(b) Classify each equilibrium.

(c) Prove the system has a homoclinic orbit.

[2 + 6 + 2 = 10 marks]

3. Consider the system

$$\begin{aligned}\dot{x} &= -x + y + xy, \\ \dot{y} &= x - y + 3y^2.\end{aligned}$$

(a) The centre manifold of the origin can be written as

$$y = \alpha x + \beta x^2 + \mathcal{O}(x^3).$$

Determine the values of α and β .

(b) On the centre manifold we have

$$\dot{x} = \gamma x^2 + \mathcal{O}(x^3).$$

Determine the value of γ .

(c) What does the stable manifold of the origin look like near the origin?

Hint: sketch a phase portrait.

[7 + 2 + 3 = 12 marks]

4. Consider the system

$$\begin{aligned}\dot{x} &= y - x^2 + \mu, \\ \dot{y} &= y(x + 2 - \mu),\end{aligned}$$

where μ is a parameter.

(a) Find all equilibria and sketch a bifurcation diagram.

(b) Identify all bifurcations seen in your diagram.

[7 + 3 = 10 marks]

Use this page if you need extra space to answer the questions. If you use it, make a note on the page of the question that you have done so, and clearly indicate here which question you are answering.

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