Random Number Generators

Random Number Generators

Random number generation is a method of producing a sequence of numbers that lack any discernible pattern.

Random Number Generators (RNGs) have applications in a wide range of different fields and are used heavily in computational simulations.

What is Random?

What do we mean by Random?

- Uncorrelated (with what you're interested in)
- Quantum systems are truly random
  (we believe)
- Some times we want repeatability from a random number generator.

Pseudo RNGs

With the exception of some specially made add-on cards, computer generated random numbers are obviously not truly random.

Computational RNGs are referred to as Pseudo Random Number Generators, meaning they are random enough.

Pseudo RNGs

PRNGs usually generate very long sequences of random numbers. For example the commonly used Mersenne Twister MT19937 has a sequence length of 2^{19937}-1.

Basically a sequence that will not recur in any practice compute time – eg the lifetime of the known universe.

Pseudo RNGs

While long sequence length is a good property of a PRNG, it is not sufficient to ensure that the numbers generated are random.

We can define a number of properties that random numbers should have and use them to test PRNGs.

Bit Properties

Ideally a generator will produce numbers that are made up of completely random bits.

From these numbers we can produce any sort of number we want.

In practice, algorithms normally generate integers. We must be careful how we transform them. Uniform floating point values don't have random bits.

Generator Algorithms

Computer systems often come supplied with a RNG built in.

eg `rand()` or `random()`

While these are fast and easy to use, they may not be very random, have short repeat sequences and may have strong correlation patterns.

Linear Congruential Generator

Many built-in RNGs use the Linear Congruential Generator or LCG. This generator is very fast, has low memory requirements but for most serious applications where randomness actually matters, it is useless.

Linear Congruential Generator

A sequence of random values $\mathbf{X_n}$ where:

$$\mathbf{X_{n+1} = (a\, X_n + c) \bmod m}$$

with well-chosen values of **a, c, m**

Period is at most **m** (and can be shorter for "bad" choices..)

Linear Congruential Generator

Example Sequences:

m=10, a=2, c=1

1
3 (2*1 + 1 % 10)
7 (2*3 + 1 % 10)
5 (2*7 + 1 % 10)
1 (2*5 + 1 % 10)
3 (2*1 + 1 % 10)
7 (2*3 + 1 % 10)
5 (2*7 + 1 % 10)
…

m=10, a=1, c=7

1
8 (1*1 + 7 % 10)
5 (1*8 + 7 % 10)
2 (1*5 + 7 % 10)
9 (1*2 + 7 % 10)
6 (1*9 + 7 % 10)
3 (1*6 + 7 % 10)
0 (1*3 + 7 % 10)
…

Linear Congruential Generator

Various sources use different parameters for the LCG:

Numerical Recipes

$m = 2^{32}$	a = 1664525	c = 1013904223

GCC

$m = 2^{32}$	a = 1103515245	c = 12345

MMIX

$m = 2^{64}$	a = 6364136223846793005	c = 1442695040888963407

Lagged Fibonacci Generator

The Lagged Fibonacci Generator is based on a generalisation of the Fibonacci sequence

1, 1, 2, 3, 5, 8, 13, 21 ....

Obtained from the sequence

$X_n = X_{n-1} + X_{n-2}$

Lagged Fibonacci Generator

The LFG generator is defined by:

$X_n = (X_{n-j} \text{ OP } X_{n-k}) \bmod m$

Where:

OP is some binary operator +, -, *, /, XOR.

$0 < j < k$

This algorithm produces a higher quality of random numbers but requires the storage of **k** past states.

Mersenne Twister

The Mersenne Twister is a widely used PRNG, the name comes from the period length which is chosen to be a Mersenne Prime.

The commonly used MT19937 algorithm stores and generates 624 at a time and can extract them one by one.

See:

http://www.math.sci.hiroshima-u.ac.jp/~m-mat/MT/emt.html

Distributions

PRNGs usually produce random unsigned integers which can easily be converted to a uniform distribution [0,1).

What about other distributions?

Gaussian or Normal distribution is common.

(Box-Muller transformation can transform a pair of uniform deviates into a Gaussian pair).

Poissonian?

Binomial?

Software Practicalities

Applications that require high quality random number generators often require a large number of them.

Correlation isn't much of a problem if there aren't many numbers that could be correlated. This leads to a number of tradeoffs between using high-quality PRNGs and fast PRNGs.

Software Practicalities

What type of data primitives does the generator use?

32-bit or 64-bit integers?

32-bit or 64-bit floating point?

What does the CPU support?

Software Practicalities

How much storage does the PRNG require?

Does the entire generator need to be in memory at once, ie does it fit in the cache?

Do we need to save/reload the generator?

Software Practicalities

How dependent is the generator on the initialisation?

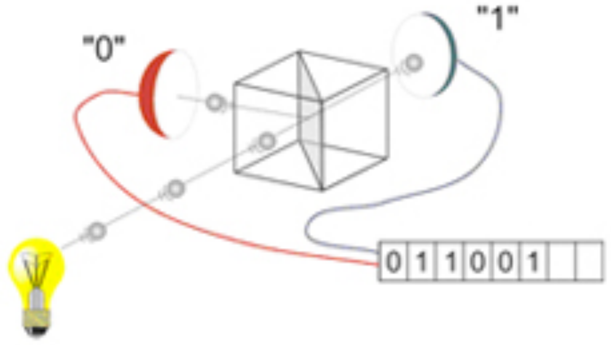
How can we generate a random initial state that will still produce high-quality random numbers?

Truly Random Numbers

While most computers are incapable of producing truly random numbers there are some modern devices capable of generating a truly random number.

Truly Random Numbers

The Quantis card is a physical random number generator that exploits a quantum optics process.



Truly Random Numbers

A simulation that uses a truly random number is not repeatable.

These quantum random number generators are not seeded and each simulation will be different.

Any interesting occurrence or phenomena will not necessarily be reproducible.

Randomness Tests

There are a number of different randomness tests that can be used to evaluate a sequence of random numbers.

Most assume the random numbers generated have a uniform distribution as other distributions are generally transformed from a uniform distribution.

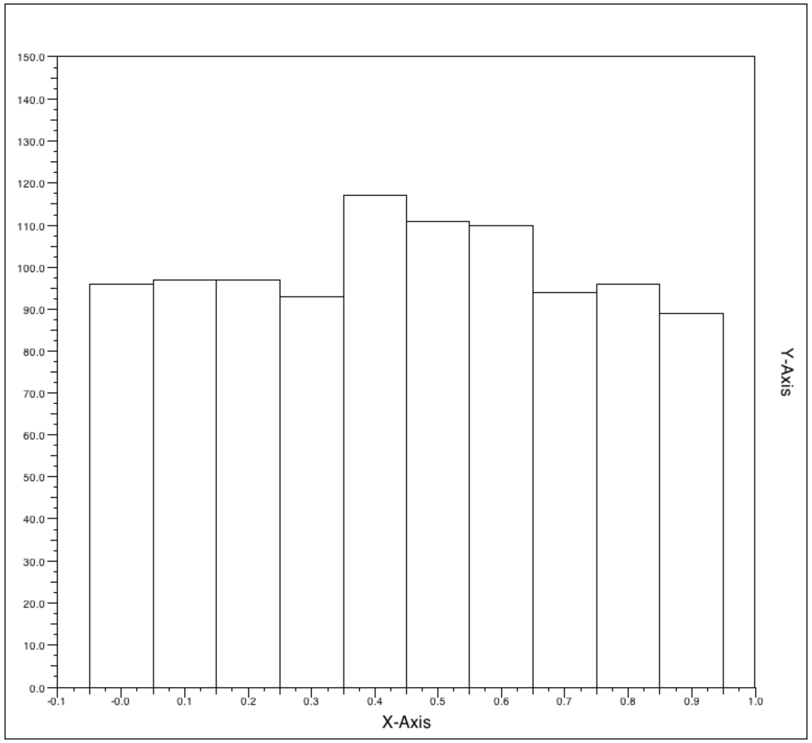
Randomness Tests

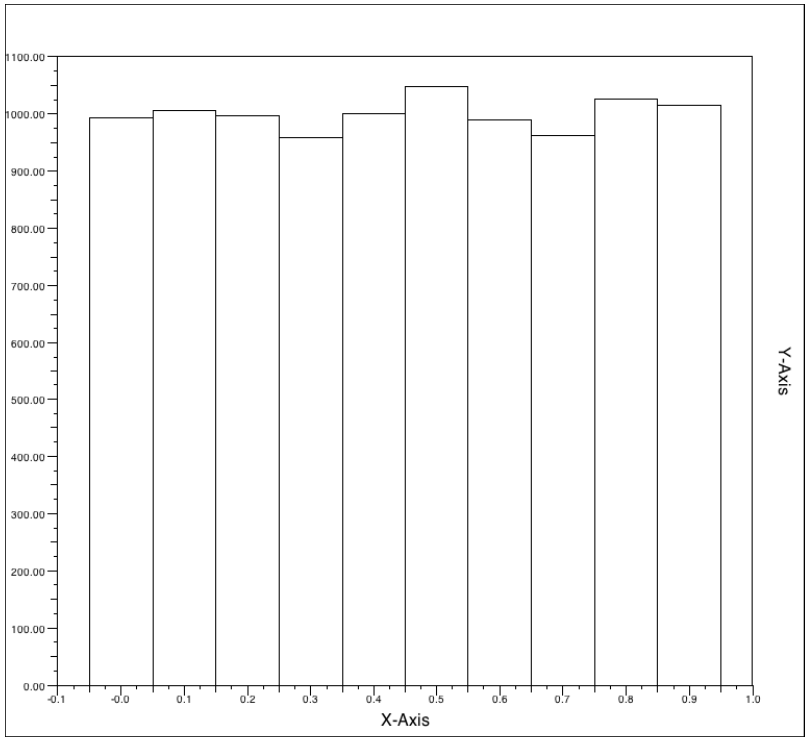
One simple method to evaluate random number generators is a histogram test.

Any sequence of uniform random numbers should produce an approximately flat histogram. The larger the number of random numbers tested, the flatter we expect the histogram to be.

Randomness Tests

1,000 samples vs 10,000 samples





Randomness Tests

This test doesn't take the sequence of the random numbers into account, or test the quality of the numbers within the histogram bins.

For example, the sequence

0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9

Has a perfectly flat histogram but obviously isn't a random sequence.

Diehard Tests

The diehard tests are a set of statistical tests with known solutions. When a random number generator is used, it should reproduce these solutions.

If the results produced by the random number generators are too different from the known solution, we have found a problem with our generator.

Diehard Tests

Parking lot test:

Randomly place unit circles in a 100x100 square. If a circle overlaps an existing one, try again. After 12,000 tries the number of ‘parked’ circles should follow a certain normal distribution.

Diehard Tests

Craps test:

The craps test: play 200,000 games of craps, counting the wins and number of throws per game. Each count should follow a given distribution.

Diehard Tests

Overlapping permutations test:

The Overlapping Permutations test: Analyze sequences of give consecutive random numbers. The 120 possible orderings should occur with statistically equal probability.

More Information

Numerous articles by G. Marsaglia, P. L'Ecuyer, M. Matsumoto and many others..

The Art of Computer Programming – Vol 2.

– D. Knuth

And of course – the internet