

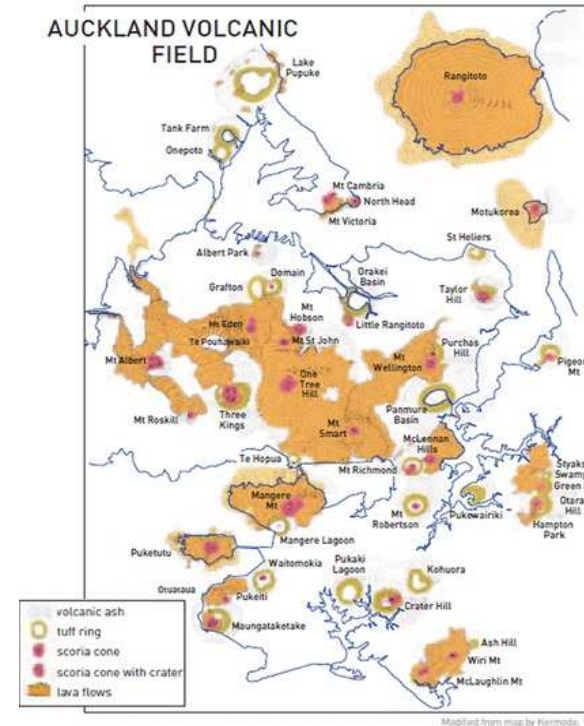
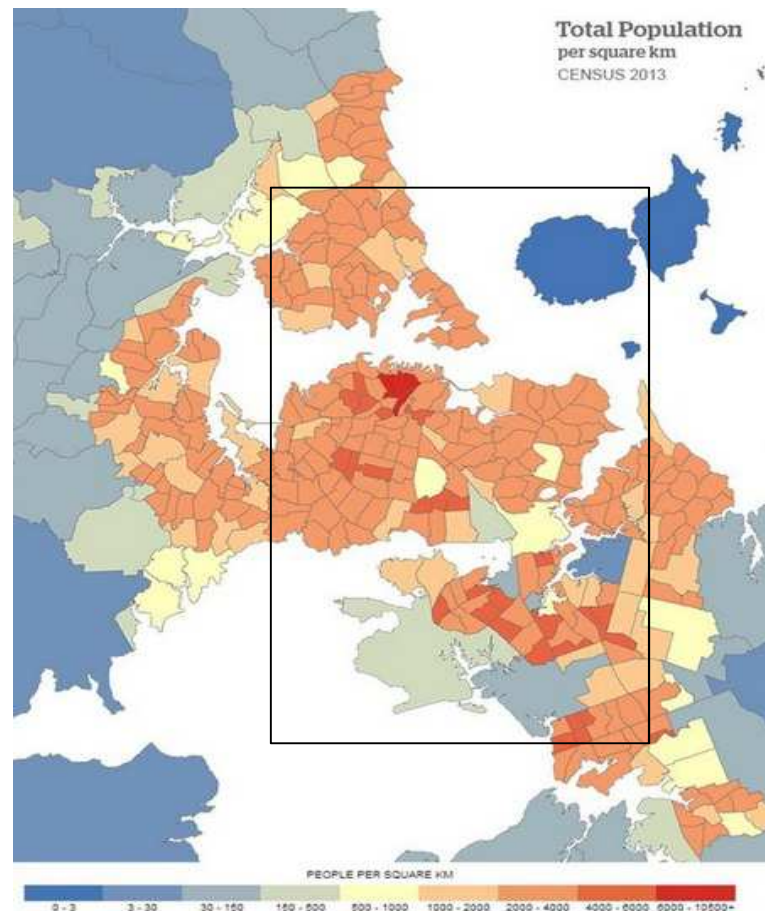


Constructing a temporal eruption record for the Auckland Volcanic Field via Bayesian age reconciliation

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The Auckland Volcanic Field



High population density, lifelines narrowly constrained

- WHERE is the next eruption likely to be?

Monogenetic volcanic fields have multiple volcanoes; a new eruption is expected to create a new volcano. Events are infrequent.

For land-use and emergency planning purposes:

where is the next eruption likely to be?

In short time frame, answered(?) by monitoring data (seismicity, gas, ...)

What about in a period of repose?

Probability forecast: estimate the *hazard* $\lambda(x)$ such that the probability of an event in the neighbourhood of x , (i.e., $y: ||y-x|| < \Delta x$) $\sim \lambda(x) \pi (\Delta x)^2$

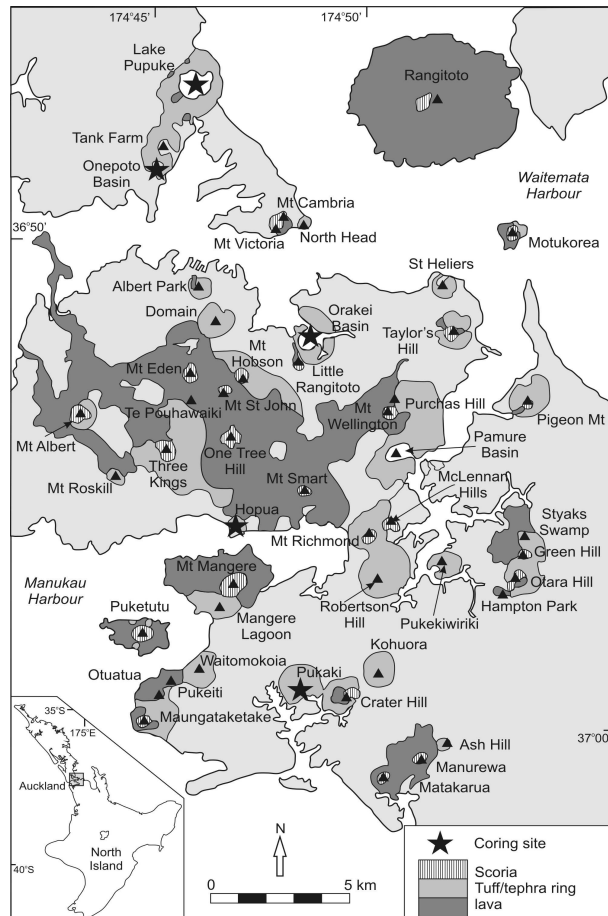
We know the spatial locations of (most) vents

Look for spatial patterns (events are more likely to occur 'near' previous events?)

BUT! Want PRESENT DAY hazard

Does the pattern change over time?

The Auckland Volcanic Field



51(?) small basaltic volcanoes

young (~250,000 years)

Most recent eruption ~600 years ago

Data:

- Stratigraphy, ~33 vents constrained in at least one direction
- Age determinations
 - Paleomagnetism ~5+ vents
 - C14, ~13 vents
 - Tephrostratigraphy, 22+ tephra in 5 locations
 - Ar-Ar, ~4 vents
 - Thermoluminescence, 2 vents
 - K-Ar, unreliable due to excess Ar
- Relative geomorphology or weathering

Decreasing reliability

Also: known vent locations, reasonable volume data
(Allen and Smith 1994; Kereszturi et al. 2013)

By reverse engineering the tephra dispersal, Bebbington and Cronin (2011, 2012) constructed *feasible* age-orderings.

Laschamp
magnetic excursion

No **apparent spatio-temporal structure**, but plenty of both temporal and spatial structure

Mono Lake magnetic
excursion

But can we find the **MOST LIKELY** age-ordering?

Name	Mean Age (ka)	Age Error (ka)	Min Order	Max Order
Onepoto Basin	248.4	27.8	1	7
Albert Park	229.8	39.5	1	7
.....
St Heliers	185.0	52.8	2	9
Te Pouhawaiki	152.9	70.3	1	34
.....
Mt St John	54.8	4.6	10	13
Maungataketake	41.4	0.4	13	15
Otuataua	41.4	0.4	14	16
McLennan Hills	40.1	1.2	13	16
One Tree Hill	34.9	0.7	16	18
.....
Hopua Basin	32.3	0.4	19	26
Puketutu	31.9	0.3	22	27
Wiri Mountain	31.9	0.3	21	28
Mt Richmond	31.7	0.3	21	28
Taylors Hill	31.7	0.3	21	28
Crater Hill	31.6	0.3	23	28
North Head	31.2	0.1	27	29
.....

Maar Tephra Thicknesses

Figure 3. Stratigraphy of post-29 cal ka BP tephra layers. Dotted lines show the tephra correlations from Molloy et al. (2009). Dashed lines show the correlations returned by our procedure when the non-linearity constant defined in Eq. (1) equals 0.5 ka. Solid lines show correlations consistent between our arrangement and that of Molloy et al. (2009).

Molloy et al. (2009), Green et al. (2014)

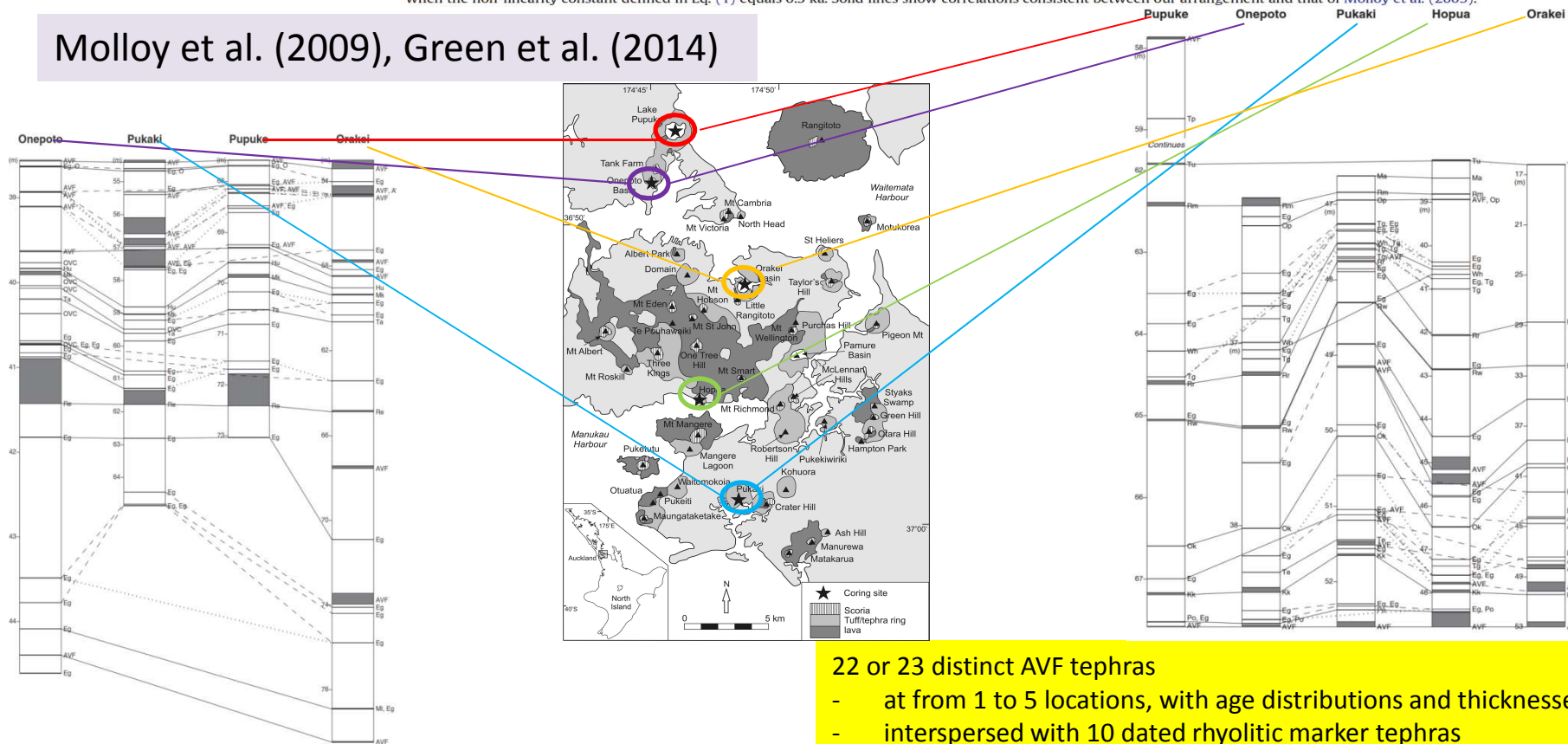
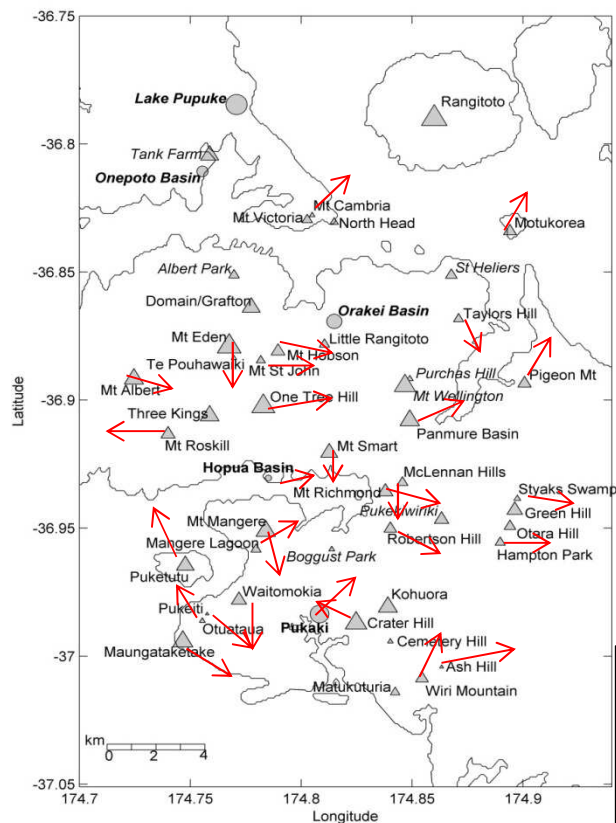
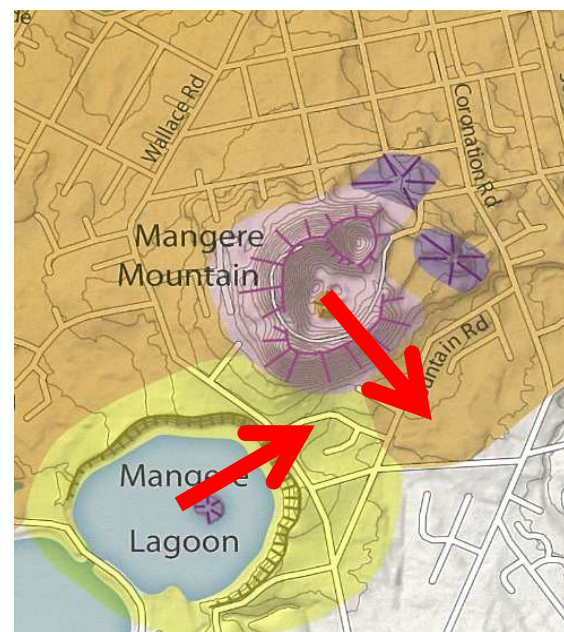


Figure 4. Stratigraphy of pre-29 cal ka BP tephra layers. Dotted lines show the tephra correlations from Molloy et al. (2009). Dashed lines show the correlations returned by our procedure when the non-linearity constant defined in Eq. (1) equals 0.5 ka. Solid lines show correlations consistent between our arrangement and that of Molloy et al. (2009).

Eruptive Tephra Volumes and Directions



Bold = cored maars
Italics = too old
Symbol size \sim volume^{1/3}
Arrow = dispersal axis



Dispersal axis in line with highest point on the rim

Volumes: Allen and Smith (1994), Kereszturi et al (2013)

Directions: Hayward et al. (2011)

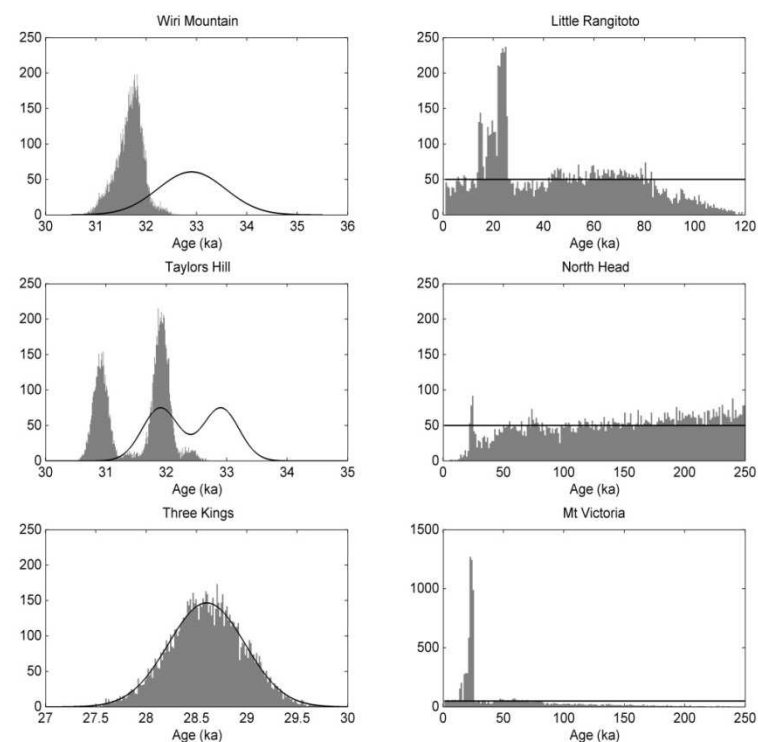
Age determinations

Volcano	Age (ka)	Dating method	References
Ash Hill	31.80 ± 0.16	¹⁴ C	Hayward (2008a)
Boggust Park	> 130	Sea level	Hayward et al. (2011a)
Crater Hill	33.33 ± 0.67	¹⁴ C	Searle (1965)
Domain	> 60	¹⁴ C	Grenfell and Kenny (1995)
Green Hill	199.83 ± 8.98	¹⁴ C methanol	Sameshima (1990)
Hampton Park	26.60 ± 8.10	⁴⁰ Ar/ ³⁹ Ar	Cassata et al. (2008)
Hopua	> 29	Oldest tephra	Molloy et al. (2009)
	< 33	Age of lava at base	Lindsay and Leonard (2009)
Kohuora	34.02 ± 0.27	¹⁴ C	Searle (1965); Grant-Taylor and Rafter (1971)
McLennan Hills	42.60 ± 3.80	⁴⁰ Ar/ ³⁹ Ar	Cassata et al. (2008)
Maungataketake	39.99 ± 0.53	¹⁴ C	Fergusson et al. (1959); Grant-Taylor and Rafter (1963); Polach et al. (1969); McDougall et al. (1969)
Motukorea	> 7	Sea level	Bryner (1991)
Mt. Albert	> 30	¹⁴ C	Fergusson et al. (1959); Grant-Taylor and Rafter (1963)
Mt. Eden	28.39 ± 0.35	¹⁴ C	East and George (2003)
Mt. Mangere	21.94 ± 0.40	¹⁴ C	Searle (1959, 1965); Grant-Taylor and Rafter (1971)
Onepoto Basin	> 99.5	Oldest tephra	Molloy et al. (2009); Green et al. (2014)
Orakei Basin	> 83.1	Oldest tephra	Molloy et al. (2009)
	< 120	Sea level	Hayward, pers. comm.
Panmure Basin	31.73 ± 0.17	¹⁴ C	Fergusson et al. (1959); Grant-Taylor and Rafter (1963); Polach et al. (1969); McDougall et al. (1969)
Pukaki	> 67	Oldest tephra	Molloy et al. (2009)
Puketutu	33.60 ± 3.70	⁴⁰ Ar/ ³⁹ Ar	Cassata et al. (2008)
Pukewairiki	> 130	Sea level	Lindsay and Leonard (2009)
Pupuke	207 ± 6	⁴⁰ Ar/ ³⁹ Ar	Cassata et al. (2008)
Roberston Hill	29.90 ± 0.60	¹⁴ C	Sandiford et al. (2002)
Three Kings	28.59 ± 0.38	¹⁴ C	Eade (2009)
Wiri Mountain	32.88 ± 0.67	¹⁴ C	Searle (1965); Grant-Taylor and Rafter (1971)

Stratigraphy

Stratigraphy	References
Ash Hill > Wiri Mountain	
Green Hill > Styaks Swamp	Sibson (1968)
Kohuora > Crater Hill	
Mangere Lagoon > Mt Mangere	Kernode and Heron (1992)
Mt Albert > Mt Roskill	Searle (1962)
Mt Eden > Mt Hobson	
Mt Mangere > Mt Smart	Kernode and Heron (1992)
Mt Roskill > Three Kings	Allen and Smith (1994)
Mt St John > Three Kings	Eade (2009)
North Head > Mt Victoria	
One Tree Hill > Hopua	Allen and Smith (1994)
One Tree Hill > Mt Eden	
One Tree Hill > Mt Mangere	Hayward (2008b)
One Tree Hill > Mt Smart	Searle (1962); Hayward (2008b)
One Tree Hill > Three Kings	Hayward (2008b)
Orakei Basin > Little Rangitoto	Kernode et al. (1992)
Pukeiti > Otutaua	Searle (1959)
Te Pouhawaiki > Mt Eden	Affleck et al. (2001)
Waitomokia > Pukeiti	Searle (1959)
Wiri Mountain > Matukutureia	
Cemetery Hill > Crater Hill	Nemeth, pers. comm.
Otara Hill > Hampton Park	Cassidy and Locke (2010)
Maungataketake > Otutaua	Cassidy and Locke (2010)
Mt. Cambria = Mt Victoria	

'Known' ages and stratigraphy combined with paleomagnetic (e.g. Mono Lake) constraints to simulate 10,000 feasible age sequences



Tephra attenuation

Expected thickness, with wind

wind speed

dispersal axis

column height term

$$\hat{T}_{ik}(r_{ik}, \xi_{ik}) = \gamma_i \exp\{-(\beta U) r_{ik} [1 - \cos(\xi_{ik} - \phi_i)]\} r_{ik}^{-\alpha}$$

without wind

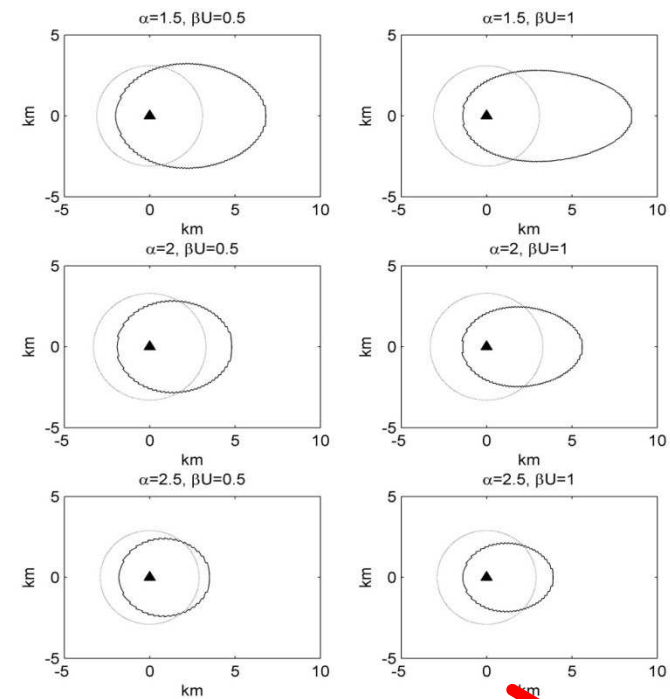
$$\hat{T}_{ik}(r_{ik}, \xi_{ik}) = \gamma_i r_{ik}^{-\alpha}$$

distance

azimuth

Gonzalez-Mellado & De la Cruz Reyna (2010) ; Kawabata et al. (2013)

Actual thickness has lognormal distribution with given mean and coefficient of variation 0.5



Wind direction

Includes terms for **observed thickness**, **not observed (but should have been!) thicknesses**, **difference in ages**, and **the presence of rhyolitic marker tephras** ...

$$u_{ijk}(T_{jk}) = \begin{cases} \frac{1}{T_{jk} \sqrt{2\pi\sigma_N^2}} \exp \left[-\frac{(\log T_{jk} - \mu_{N_{ik}})^2}{2\sigma_N^2} \right], & T_{jk} > 0 \\ \frac{1}{\sqrt{2\pi\sigma_N^2}} \int_0^{0.05} \frac{1}{t} \exp \left\{ -\frac{[\log(t) - \mu_{N_{ik}}]^2}{2\sigma_N^2} \right\} dt, & T_{jk} = 0 \\ 1, & T_{jk} = \text{NA}, \end{cases}$$

$$v_{ik}^n(T_{jk}) = \begin{cases} \frac{1}{\sqrt{2\pi\sigma_N^2}} \int_0^{0.05} t^{-1} \exp \left\{ -\frac{[\log(t) - \mu_{N_{ik}}]^2}{2\sigma_N^2} \right\} dt, & s_i^n \in C_k \\ 1, & \text{otherwise}, \end{cases}$$

$$f_j^n(s_i^n) = \frac{1}{\sqrt{2\pi\sigma_j^2}} \exp \left[-\frac{(s_i^n - \mu_j)^2}{2\sigma_j^2} \right]$$

$$g_{j,j^*}^n(s_i^n) = \begin{cases} \Pr(j^* < s_i^n) = \frac{1}{\sqrt{2\pi\sigma_{j^*}^2}} \int_{-\infty}^{s_i^n} \exp \left[-\frac{(t - \mu_{j^*})^2}{2\sigma_{j^*}^2} \right] dt, & \text{AVF } j \text{ older than } j^* \\ \Pr(j^* > s_i^n) = 1 - \frac{1}{\sqrt{2\pi\sigma_{j^*}^2}} \int_{-\infty}^{s_i^n} \exp \left[-\frac{(t - \mu_{j^*})^2}{2\sigma_{j^*}^2} \right] dt, & \text{otherwise}, \end{cases}$$

$$\log L_{ij}^n = \begin{cases} \sum_{k=1}^5 \log u_{ijk}(T_{jk}) + \log f_j^n(s_i^n) + \sum_{j^*=1}^{10} \log g_{j,j^*}^n(s_i^n) & j = \{1, \dots, 20\} \\ \sum_{k=1}^5 \log v_{ik}^n(T_{jk}) & j = 0, \end{cases}$$

Solved by linear programming:

$$\log L^n = \sum_{i=1}^{41} \sum_{j=1}^{20} x_{ij}^n \left[\sum_{k=1}^5 \log u_{ijk}(T_{jk}) + \log f_j^n(s_i^n) + \sum_{j^*=1}^{10} \log g_{j,j^*}^n(s_i^n) \right] + \sum_{i=1}^{41} \sum_{k=1}^5 x_{i0}^n \log v_{ik}^n(T_{jk}).$$

with constraints for stratigraphy, etc.

$$\sum_{i=1}^{41} (ix_{i,j+1}^n - ix_{ij}^n) > 0, \forall j = 1, \dots, 19.$$

Table 6: Marginal posterior probabilities for a volcano to have produced a given tephra for the baseline scenario with $\alpha = 2.0$, $\beta U = 0.5$. The global best arrangement is indicated by bold type.

Volcano	AVF0	AVF1	AVF2	AVF3	AVF4	AVF5	AVF6	AVF7	AVF8	AVF9	AVF10	AVF11	AVF12	AVF13	AVF14	AVF15	AVF16	AVF17	AVF18	AVF19	AVF20
Ash Hill	.8011					.0004	.1612	.0373													
Cemetery Hill	.0024				.0002	.0170	.1326	.1716	.5343	.1106	.0088										
Crater Hill						.0001	.0150	.0068	.2779	.5690	.1301	.0011									
Domain	.1320	.0870	.0218																		
Green Hill	.0102	.0007	.0031	.0162	.0255	.0160	.0050	.0003	.0004	.0037	.0376	.0164	.0555	.1900	.1223	.1029	.0775	.0874	.1098	.0878	.0308
Hampton Park	.3039					.0001								.0648	.1480	.1912	.1051	.0693	.0167	.0070	.0039
Hopua Basin	.8042					.0639	.0499	.0359	.0003	.0021	.0413	.0021									
Kohuora	.0001	.0018	.0171	.0030		.0576	.0514	.0010	.0010												
Little Rangitoto	.0410	.1987	.1023	.1444	.0215	.0117	.0013	.0009	.0008	.0025	.0133	.0056	.0182	.0497	.0383	.0288	.0208	.0317	.0488	.0530	.0737
Mangere Lagoon	.7382	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
Matakitaki	.0299									.0008	.0076	.0032	.0067	.1381	.1007	.0861	.0683	.0642	.0950	.2123	.1961
Maungataketake	.9943	.0012		.0034		.0002	.0009														
McLennan Hills	.9259		.0162			.0435	.0128	.0009	.0007												
Motukorea	.9494	.0023	.0007	.0002		.0019								.0065	.0008	.0029	.0021	.0276	.0010	.0016	.0024
Mt Albert	.8857	.0585	.0360	.0184	.0007	.0005	.0001														
Mt Cambria	.7322	.0105	.0238	.0175	.0035	.0083															
Mt Eden	.0442									.0006	.0363	.3759	.5430								
Mt Hobson	.1233									.0002	.0038	.0184		.1215	.0880	.0743	.0609	.0841	.1128	.1302	.1825
Mt Mangere	.0095													.0048	.0431	.1439	.3543	.3037	.1366	.0041	
Mt Richmond	.7783					.0013	.0717	.1487													
Mt Roskill	.9408	.0185	.0261	.0022	.0001	.0067	.0001	.0004			.0051					.0004	.0050	.0266	.0786	.2761	.3807
Mt Smart	.0099																				
Mt St John	.7185	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
Mt Victoria	.1755	.0696	.0735	.0718	.0136	.0070	.0030	.0001	.0001	.0018	.0043	.0050	.0076	.1063	.1252	.0652	.0846	.0173	.0773	.0382	.0530
North Head	.7306	.1006	.0624	.0508	.0077	.0036	.0004	.0001	.0001	.0003	.0050	.0015	.0050	.0112	.0062	.0041	.0028	.0026	.0019	.0015	.0016
One Tree Hill	.6754	.0843	.1248	.0981	.0025	.0109	.0009	.0002	.0001	.0003	.0018	.0009									
Otara Hill	.0057	.0004	.0006	.0075	.0052	.0030	.0020			.0008	.0073	.0040	.0112	.2147	.2064	.1806	.1282	.0605	.0686	.0251	.0082
Otututu	.6172	.0065	.0091	.2341	.0177	.0040	.0002	.0002													
Panmure Basin	.1300		.0001	.0004	.0004	.7081	.0807	.0508		.0004	.0231										
Pigeon Mountain	.7722	.0307	.0443	.0358	.0005	.0047					.0062			.0225	.0122	.0161	.0074	.0281	.0039	.0036	.0118
Pukeki	.1	.0886																			
Puketutu	.9270				.0006	.0033	.0383	.0496	.1623	.2245	.4942	.0002									
Robertson Hill	.9995					.0005															
Styaks Swamp	.5310													.0116	.0165	.0339	.0362	.0535	.0503	.0538	.2182
Taylor Hill	.5495			.0001		.0198	.2255	.2043			.0088										
Te Pouhawaiki	.7341	.0941	.0740	.0702	.0079	.0063	.0011	.0003	.0003	.0004	.0087	.0026									
Three Kings										.0032	.0954	.5717	.3297								
Waitomokia	.1																				
Wiri Mountain						.0023	.1535	.2843	.0224	.0783	.0647										

- Three Kings (large volume, central, good age) one of AVF 9-12
- Crater Hill (large, good age, azimuth towards maar) also always matched
- Green Hill, Little Rangitoto, Mt Victoria, North Head can match any tephra
- Pukeiti, Waitomokia don't match any tephra

Find *most likely global* arrangement with another linear program ...

Best overall arrangements

Table 7: Best global arrangement of assigning Volcano *i* to AVF tephras *j*. Scenarios are outlined in the sensitivity analysis section. The six parameter sets are (A = { $\alpha = 1.5, \beta U = 0.5$ }; B = { $\alpha = 2, \beta U = 0.5$ }; C = { $\alpha = 2.5, \beta U = 0.5$ }; D = { $\alpha = 1.5, \beta U = 1$ }; E = { $\alpha = 2, \beta U = 1$ }; F = { $\alpha = 2.5, \beta U = 1$ }). Bold type indicates scenario-determined identifications.

Tephra	Volcano	Baseline	Maungataketake and North Head older	Scenario	Three Kings is AVF9	Three Kings is AVF10	AVF9 split in two
AVF1	Little Rangitoto	ADEF		ACDEF	All		ACDEF
	Otuataua Domain	BC	DEF	B			B
AVF2	One Tree Hill	ADEF	ABC	ACDEF	All		ACDEF
	Little Rangitoto	BC	All	B			B
AVF3	Otuataua	All		All	All		All
	Little Rangitoto		All				
AVF4	Kohuora	All	All	All	All		All
AVF5	Panmure Basin	All	All	All	All		All
AVF6	Ash Hill	DEF	DEF				EE
	Taylor's Hill	ABC	ABC		DEF		ABCD
	Wiri Mountain			AB	ABC		
	Crater Hill			CDE			
	Cemetery Hill			F			
AVF7	Wiri Mountain	All	All	CDEF	EF		BCDEF
	Cemetery Hill			AB	ABCD		A
AVF8	Cemetery Hill	All	All				BCDEF
	Crater Hill			AB	All		A
AVF9	Puketutu			CDEF			
	Crater Hill	All	All				
	Three Kings			All			
AVF9A	Puketutu				All		
	Wiri Mountain						A
AVF9B	Crater Hill						BCDEF
	Puketutu						All
AVF10	Puketutu	All	All				
	Robertson Hill			ABC			
	Hopua			DEF			All
AVF11	Three Kings	ABC	ABC		All		ABC
	Three Kings	DEF	DEF	DEF			DEF
	Mt Eden			ABC	All		
AVF12	Hopua	ABC	ABC	ABC	All		ABC
	Mt Eden	DEF	DEF				DEF
	Three Kings			DEF			
AVF13	Green Hill	All	All	ABC	All		All
	Mt Hobson			DEF			
AVF14	Otara Hill	All	All	All	All		All
AVF15	Hampton Park	All	All	All	All		All
AVF16	Mt Mangere	All	CDEF	All	All		All
	Matukuturua		AB				
AVF17	Mt Cambria	All		BCDEF	All		All
	Mt Mangere		AB				
	Mt Hobson		CDEF	A			
AVF18	Mt Hobson	AB	AB		A		AB
	Mt Smart	CDEF	CDEF	All	BCDEF		CDEF
AVF19	Mt Smart	AB	AB		A		AB
	Matukuturua	CDEF	CDEF	All	BCDEF		CDEF
AVF20	Styaks Swamp	All	All	All	All		All

Scenarios

- Maungataketake $\sim 87.4 \pm 2.4$ ka, North Head > 80 ka
- Three Kings is AVF 9 or AVF 10
- AVF 9 (thick in N and S, thin in middle) is actually 2 distinct tephras

Mono Lake Excursion

6 Conditions

Wind: DEF $>$ ABC

Column height: AD $>$ BE $>$ DF

Results do not change if CV is doubled to better represent certain massively thick tephras.

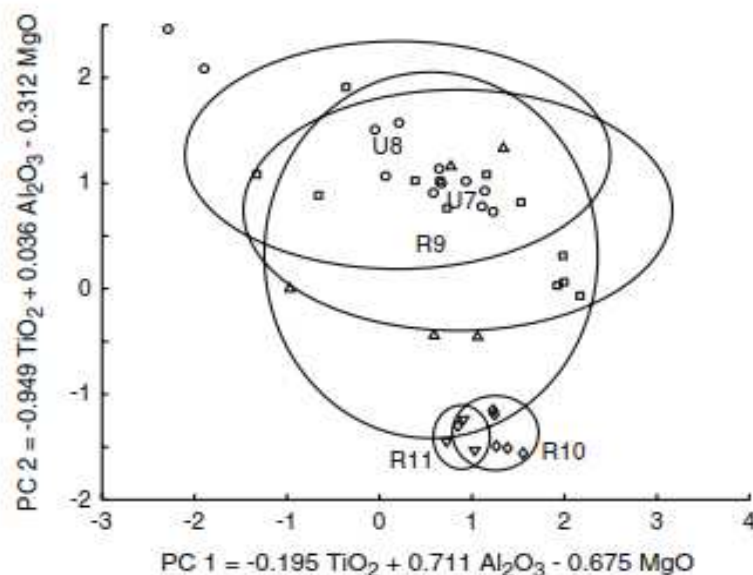
Table 8: Comparison of event order models

Tephra	Baseline model, this work	Bebbington and Cronin (2012)
AVF1	Little Rangitoto/Domain	Pukaki
AVF2	One Tree Hill/Little Rangitoto	Domain
AVF3	Otuataua	Mt St John
AVF4	Kohuora	One Tree Hill
AVF5	Panmure Basin	Motukorea
AVF6	Taylors Hill / Ash Hill	Kohuora
AVF7	Wiri Mountain	Crater Hill
AVF8	Cemetery Hill	Puketutu
AVF9	Crater Hill	Hopua Basin
AVF10	Puketutu	North Head
AVF11	Three Kings / Mt Eden	Panmure Basin
AVF12	Three Kings / Mt Eden	Three Kings
AVF13	Green Hill	Mt Eden
AVF14	Otara Hill	Mt Hobson
AVF15	Hampton Park	Little Rangitoto
AVF16	Mt Mangere	Pigeon Mountain
AVF17	Mt Cambria	Mangere Lagoon
AVF18	Mt Hobson / Mt Smart	Mt Mangere
AVF19	Mt Smart / Matakutereia	Mt Smart
AVF20	Styaks Swamp	Styaks Swamp

- Taylors Hill was most likely the first of the Mono Lake volcanoes to erupt.
- Almost all of the large volume volcanoes are allocated.
 - Maungataketake and Mt Roskill have unfavourable directions.
 - Mt Albert has large volume and good dispersal axis, but only age data is > 35 ka

Incorporate geochemistry of sources and deposits

- Additional likelihood penalty if the source and deposit geochemistry differs



Describe multi-element geochemistry by multivariate Gaussian distribution (e.g., in 2-D an ellipse)

- Can use principle components to reduce dimensionality
- Need data to quantify whatever systematic bias and heteroscedasticity exists between lava/scoria/tuff and tephra geochemistry.

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