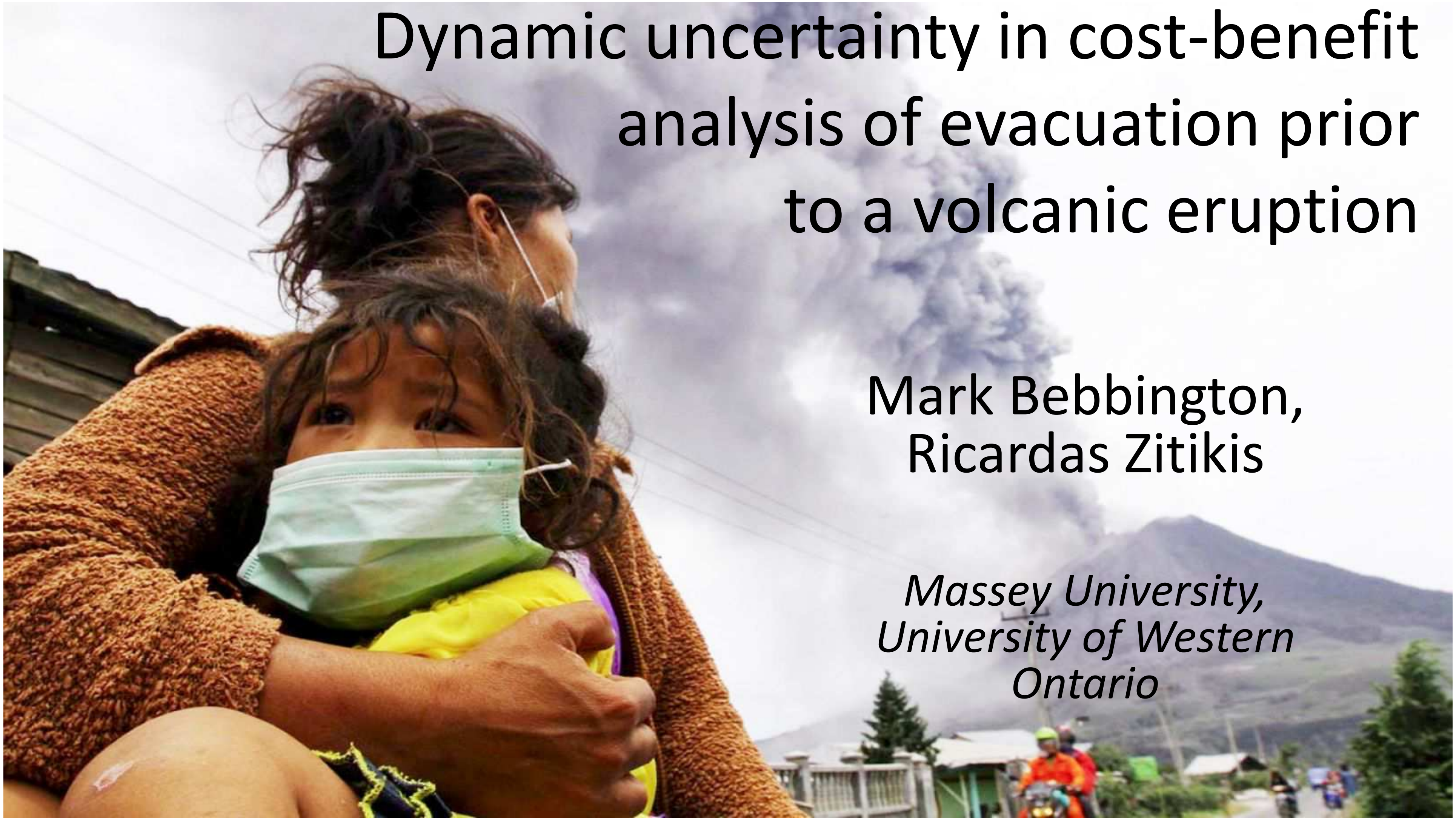

Dynamic uncertainty in cost-benefit analysis of evacuation prior to a volcanic eruption

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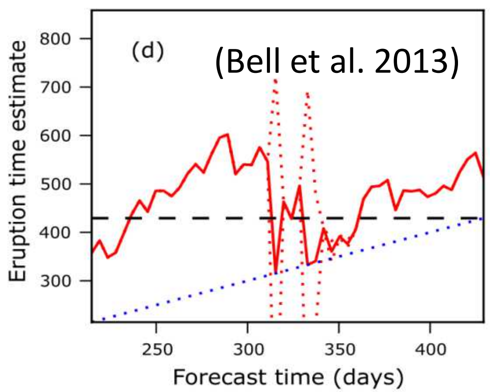
Short-term Eruption Forecasting

Many forecasting methods. Inputs: physical monitoring data
Outputs: either predict onset date (with error),
or probability of eruption in fixed time window

Forecast Failure Method
(Voight 1988; …)

(earthquake counts)



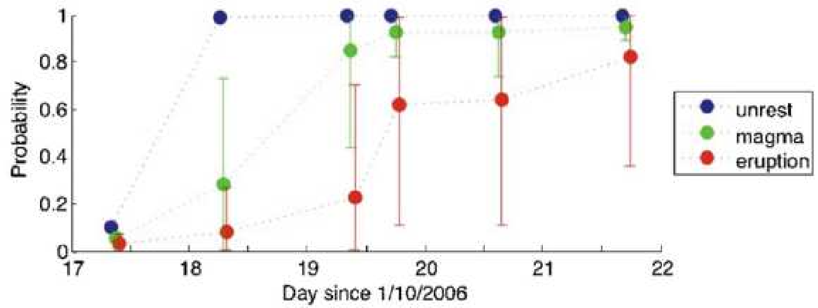
Bayesian Event Tree
(Marzocchi et al. 2008)

Table 2 Monitoring parameters (column 1), order relationship (rel.), lower and upper thresholds and relative units (column 2), and measured values as in MESIMEX real time bulletins (columns 3–8) relative to node 1 for Vesuvius

Node 1

Parameter	Order rel., thresholds and units	Bulletin 1 Oct 17, 9 am	Bulletin 2 Oct 18, 7 am	Bulletin 3 Oct 19, 9 am	Bulletin 4 Oct 19, 6 pm	Bulletin 5 Oct 20, 3 pm	Bulletin 6 Oct 21, 5 pm
n_e monthly number of seismic events with $M_d \geq 1.9$ at OVO station	>23; 150 month^{-1}	10	38	61	104	183	258
M_d monthly largest duration magnitude of seismic events at OVO station	>3.4; 4.3 month^{-1}	2.8	4.2	4.2	4.2	4.2	4.2
n_{LF} monthly number of LF scriptsize events deeper than 1 km	>1; 3 month^{-1}	0	0	2	26	61	131
Π_{SO_2} presence of significant SO_2 (1=yes)	=1	0	0	1	1	1	1
Φ_{CO_2} daily CO_2 emission rate	>5; 30 kg m^{-2} day^{-1}	10	20	20	30	300	400
$\dot{\varepsilon}$ strain rate (inflation)	>0; 0 day^{-1}	0	0	0	5 10^{-5}	5 10^{-5}	2 10^{-4}
T temperature of the fumaroles in the crater (°C)	>98; 105	95	95	100	110	110	110

(*lots* of data)



How to translate forecasts into (in)action?

Probability of eruption method lends itself to cost-benefit analysis:

P = probability of eruption
L = cost (deaths etc.) from eruption if no evacuation
E = cost of evacuation

→ Evacuate if P > E/L (*)

This has many advantages: it is easy for the scientist to explain to the decision-maker,
it is easy for the scientist to explain to the decision-maker,
it is easy for the scientist to explain to the decision-maker,
it is easy for the scientist to explain to the decision-maker …

However, the calculations leading to (*) are static, ignoring:

- evacuation time
- time to eruption
- spontaneous self-evacuation and returns
- the possible qualms of the decision-maker (‘cry-wolf’, litigation)

How much does (*) change if we try including these factors?

Length of alert window

Use BET_EF as an example

Typically we have a series of bulletins. For how long are these informative?

- Let F(u) be the estimated probability of an eruption in time window $(t_0, t_0 + u)$, based on a bulletin at time t_0.

Then the end of the *alert period* is: $t_{max} = \inf\left\{t > s : \sup_{u>t}\left[F(t_0 + u + s) - F(t_0 + u)\right] < q_0\right\}$

(the first time the future (s =) 30 day window probability drops below q_0)

q_0 = 0.01 per 30 days

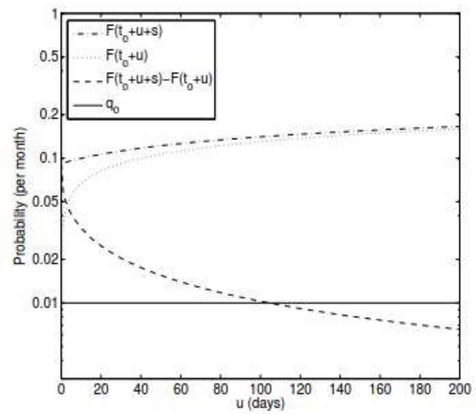


Fig. 1 Calculating the end of the alert period t_{max}. When the probabilities at each end of a moving window of length s days differ by less than q_0, the alert period ends.

Costs and other parameters

M = *relative* (to a reference event of M=1) intensity of the hazard
$C(t)$ = economic disruption cost incurred in time t
$L_M(t)$ = cost of fatalities if evacuation is called time t before eruption (intensity M)
$D_M(t)$ = cost of displacement if evacuation is called time t before eruption (intensity M)

The reference event kills a proportion δ

$$L_M(t) = \delta c M e^{-t/\eta}$$

Table 1 Costs during the alert period $(t_0, t_0 + t_{\max}]$

	State of Nature	
Actions	No eruption $(T > t_{\max})$	Eruption occurs $(T \le t_{\max})$
No evacuation	0	$L_M(0) + D_M(0) + B$
Evacuation called	$C(t_{\max} - \tau) + A$	$C(T-\tau) + L_M(T-\tau) + D_M(T-\tau) + a$

A and B are 'political' costs

$$C(t) = b\int_0^t e^{-s/\rho}ds = b\rho\left(1 - e^{-t/\rho}\right)$$

Evacuees 'return' at random, after an average time ρ

$$D_M(t) = dM\left(1 - \delta M\left(e^{-t/\eta} + (1 - e^{-t/\eta})e^{-t/\rho}\right)\right)$$

Evacuation takes a random time, average η

Per-capita costs: a (evacuation cost), b (economic dislocation), c (death), d (displacement)

Evacuation Criterion

Cost-benefit analysis:

- During the alert period $(t_0, t_0 + t_{max}]$, an evacuation is justified at time $t_0 + \tau$ if the expected cost is less than it would be under no evacuation.
  - The set of all justifiable evacuation times is the *evacuation window*.
  - The lower boundary of the evacuation window is thus the point at which the evacuation order is issued.

Time to eruption onset:

T = eruption onset, T ~ Gamma(κ,θ) random variable: $$f_T(t) = \frac{t^{\kappa-1}\exp(-t/\theta)}{\theta^{\kappa}\Gamma(\kappa)}$$

- allows for over- (or under-) dispersion

The simple model gives a 'simple' result:

Theorem 1 *Criterion 1 for evacuation at time* $t_0 + \tau$ *is satisfied if and only if* $\tau \le t_{\max}$ *is such that*

$$\Delta(2,1) + \Delta(2,2) < \Delta(1,2),$$

where

$$\Delta(2,1) = \left(b\rho - b\rho e^{-(t_{\max}-\tau)/\rho} + \mathbf{E}[A]\right) \frac{\Gamma(\kappa) - \gamma(\kappa, t_{\max}/\theta)}{\Gamma(\kappa) - \gamma(\kappa, \tau/\theta)}$$

is the expected cost under evacuation but no eruption during the alert period,

$$\begin{aligned}\Delta(2,2) = &\left(b\rho + d\mathbf{E}[M] + a\right) \frac{\gamma(\kappa, t_{\max}/\theta) - \gamma(\kappa, \tau/\theta)}{\Gamma(\kappa) - \gamma(\kappa, \tau/\theta)} \\ &+ \frac{(b\rho + d\delta\mathbf{E}[M^2])e^{\tau/\rho}}{\theta^\kappa \omega^\kappa} \frac{\gamma(\kappa, t_{\max}\omega) - \gamma(\kappa, \tau\omega)}{\Gamma(\kappa) - \gamma(\kappa, \tau/\theta)} \\ &+ \frac{\delta(c\mathbf{E}[M] - d\mathbf{E}[M^2])e^{\tau/\eta}}{\theta^\kappa \chi^\kappa} \frac{\gamma(\kappa, t_{\max}\chi) - \gamma(\kappa, \tau\chi)}{\Gamma(\kappa) - \gamma(\kappa, \tau/\theta)} \\ &+ \frac{d\delta\mathbf{E}[M^2]e^{\tau/\zeta}}{\theta^\kappa \xi^\kappa} \frac{\gamma(\kappa, t_{\max}\xi) - \gamma(\kappa, \tau\xi)}{\Gamma(\kappa) - \gamma(\kappa, \tau/\theta)}\end{aligned}$$

is the expected cost under eruption and evacuation during the alert period, and

$$\Delta(1,2) = \left((\delta c + d)\mathbf{E}[M] - d\delta\mathbf{E}[M^2] + \mathbf{E}[B]\right) \frac{\gamma(\kappa, t_{\max}/\theta)}{\Gamma(\kappa)}$$

is the expected cost under no evacuation, where $\omega = 1/\rho + 1/\theta, \chi = 1/\eta + 1/\theta, \xi = 1/\eta + 1/\rho + 1/\theta$.

An elementary proof … by mathematical standards

Expected cost under no evacuation There are two possibilities: no eruption during the alert period ($T > t_{\max}$), and thus the cost is 0, or an eruption occurs ($T \le t_{\max}$), in which case the expected cost is

$$\Delta(1,2) = \mathbf{E}[L_M(0) + D_M(0) + B \mid T \le t_{\max}]\mathbf{P}[T \le t_{\max}]. \tag{15}$$

Hence $\Delta(1,2)$ is the entire expected cost when no evacuation occurs during the alert period. Since both M and B are independent of T by assumption, and given $L_M(0) = \delta c M$ and $D_M(0) = dM(1-\delta M)$, the expectation on the right-hand side of Eq. (15) is equal to $(\delta c + d)\mathbf{E}[M] - d\delta\mathbf{E}[M^2] + \mathbf{E}[B]$. Furthermore, the probability $\mathbf{P}[T \le t_{\max}] = F_T(t_{\max}) = \gamma(\kappa, t_{\max}/\theta)/\Gamma(\kappa)$ by Eq. (2). Thus

$$\Delta(1,2) = \Big((\delta c + d)\mathbf{E}[M] - d\delta\mathbf{E}[M^2] + \mathbf{E}[B]\Big)\frac{\gamma(\kappa, t_{\max}/\theta)}{\Gamma(\kappa)}.$$

Expected cost under evacuation at τ, assuming $\tau \le \min\{T, t_{\max}\}$, where we use $\tau \le T$ as a short-hand, keeping in mind that $\tau < t_{\max}$ here.

This expected cost is the sum of two costs: the expected cost

$$\Delta(2,1) = \mathbf{E}[C(t_{\max} - \tau) + A \mid T > t_{\max}, \tau \le T]\mathbf{P}[T > t_{\max} \mid \tau \le T]. \tag{16}$$

when there is no eruption during the alert period, and the expected cost

$$\Delta(2,2) = \mathbf{E}[C(T-\tau) + L_M(T-\tau) + D_M(T-\tau) + a \mid T \le t_{\max}, \tau \le T]\mathbf{P}[T \le t_{\max} \mid \tau \le T]. \tag{17}$$

when there is an eruption during the alert period.

Expected cost $\Delta(2,1)$ The condition in the expectation reduces to $T > t_{\max}$ because $\tau \le t_{\max}$. Furthermore, $C(t_{\max} - \tau)$ is not random and the additional cost A is assumed to be independent of T. Hence, the expectation on the right-hand side of Eq. (16) reduces to $C(t_{\max} - \tau) + \mathbf{E}[A]$. As to the probability, we have

$$\begin{aligned}\mathbf{P}[T > t_{\max} \mid \tau \le T] &= \frac{\mathbf{P}[T > t_{\max}]}{\mathbf{P}[T > t_{\max}] + \mathbf{P}[\tau \le T \le t_{\max}]}\\ &= \frac{1 - F_T(t_{\max})}{1 - F_T(\tau)},\end{aligned}$$

where we used the continuity of the cdf F_T to write $\mathbf{P}[\tau \le T < \infty]$ as $1 - F_T(\tau)$. Hence,

$$\begin{aligned}\Delta(2,1) &= \Big(C(t_{\max} - \tau) + \mathbf{E}[A]\Big)\frac{1 - F_T(t_{\max})}{1 - F_T(\tau)}\\ &= \Big(b\rho - b\rho e^{-(t_{\max}-\tau)/\rho} + \mathbf{E}[A]\Big)\frac{\Gamma(\kappa) - \gamma(\kappa, t_{\max}/\theta)}{\Gamma(\kappa) - \gamma(\kappa, \tau/\theta)}.\end{aligned}$$

where we have used Eq. (4) for $C(t)$ and Eq. (2) for the cdf $F_T(t)$.

Expected cost $\Delta(2,2)$ Starting with the probability on the right-hand side of Eq. (17),

$$\begin{aligned}\mathbf{P}[T \le t_{\max} \mid \tau \le T] &= \frac{\mathbf{P}[T \le t_{\max}, \tau \le T]}{\mathbf{P}[T \le t_{\max}, \tau \le T] + \mathbf{P}[T > t_{\max}]}\\ &= \frac{F_T(t_{\max}) - F_T(\tau)}{1 - F_T(\tau)}\\ &= \frac{\gamma(\kappa, t_{\max}/\theta) - \gamma(\kappa, \tau/\theta)}{\Gamma(\kappa) - \gamma(\kappa, \tau/\theta)},\end{aligned}$$

keeping in mind that $\tau \le t_{\max}$. Hence,

$$\begin{aligned}\Delta(2,2) = \Big(&\mathbf{E}[C(T-\tau) \mid \tau < T \le t_{\max}] + \mathbf{E}[L_M(T-\tau) \mid \tau < T \le t_{\max}]\\ &+ \mathbf{E}[D_M(T-\tau) \mid \tau < T \le t_{\max}] + a\Big)\frac{\gamma(\kappa, t_{\max}/\theta) - \gamma(\kappa, \tau/\theta)}{\Gamma(\kappa) - \gamma(\kappa, \tau/\theta)}.\end{aligned} \tag{18}$$

We next evaluate the three expectations on the right-hand side of Eq. (18), calling them C-, L- and D-based expectations.

C-based expectation Using Eq. (4) for $C(t)$, we have

$$\begin{aligned}\mathbf{E}[C(T-\tau) \mid \tau < T \le t_{\max}] &= \frac{1}{F_T(t_{\max}) - F_T(\tau)}\int_\tau^{t_{\max}} C(t-\tau)dF_T(t)\\ &= b\rho - \frac{b\rho e^{\tau/\rho}}{F_T(t_{\max}) - F_T(\tau)}\int_\tau^{t_{\max}} e^{-t/\rho} f_T(t)dt.\end{aligned} \tag{19}$$

Using Eq. (3) for the density $f_T(t)$ as well as the notation $\omega = 1/\rho + 1/\theta$, we obtain

$$\begin{aligned}\int_\tau^{t_{\max}} e^{-t/\rho} f_T(t)dt &= \frac{1}{\theta^\kappa\Gamma(\kappa)}\int_\tau^{t_{\max}} t^{\kappa-1}\exp\{-t\omega\}\,dt\\ &= \frac{1}{\theta^\kappa\Gamma(\kappa)\omega^\kappa}\Big(\gamma(\kappa, t_{\max}\omega) - \gamma(\kappa, \tau\omega)\Big).\end{aligned} \tag{20}$$

Combining Eqs. (19) and (20), we find that

$$\begin{aligned}\mathbf{E}[C(T-\tau) \mid \tau < T \le t_{\max}] &= b\rho - \frac{b\rho e^{\tau/\rho}}{\theta^\kappa\Gamma(\kappa)\omega^\kappa}\frac{\gamma(\kappa, t_{\max}\omega) - \gamma(\kappa, \tau\omega)}{F_T(t_{\max}) - F_T(\tau)}\\ &= b\rho - \frac{b\rho e^{\tau/\rho}}{\theta^\kappa\omega^\kappa}\frac{\gamma(\kappa, t_{\max}\omega) - \gamma(\kappa, \tau\omega)}{\gamma(\kappa, t_{\max}/\theta) - \gamma(\kappa, \tau/\theta)}.\end{aligned} \tag{21}$$

L-based expectation Using Eq. (5) for $L_M(t)$ and also the independence of M and T,

$$\mathbf{E}[L_M(T-\tau) \mid \tau < T \le t_{\max}] = \frac{\delta c\mathbf{E}[M]e^{\tau/\eta}}{F_T(t_{\max}) - F_T(\tau)}\int_\tau^{t_{\max}} e^{-t/\eta} f_T(t)dt. \tag{22}$$

To calculate the integral on the right-hand side of Eq. (22), we use Eq. (20) with η instead of ρ, and thus obtain

$$\begin{aligned}\mathbf{E}[L_M(T-\tau) \mid \tau < T \le t_{\max}] &= \frac{\delta c\mathbf{E}[M]e^{\tau/\eta}}{\theta^\kappa\Gamma(\kappa)\chi^\kappa}\frac{\gamma(\kappa, t_{\max}\chi) - \gamma(\kappa, \tau\chi)}{F_T(t_{\max}) - F_T(\tau)}\\ &= \frac{\delta c\mathbf{E}[M]e^{\tau/\eta}}{\theta^\kappa\chi^\kappa}\frac{\gamma(\kappa, t_{\max}\chi) - \gamma(\kappa, \tau\chi)}{\gamma(\kappa, t_{\max}/\theta) - \gamma(\kappa, \tau/\theta)},\end{aligned} \tag{23}$$

where $\chi = 1/\eta + 1/\theta$.

D-based expectation We begin by expanding the formula of $D_M(t)$ as

$$D_M(t) = dM - d\delta M^2e^{-t/\eta} - d\delta M^2e^{-t/\rho} + d\delta M^2e^{-t/\zeta},$$

where ζ is such that $1/\zeta = 1/\eta + 1/\rho$. Hence, recalling that $\chi = 1/\eta + 1/\theta$ and $\omega = 1/\rho + 1/\theta$, we have, denoting $\xi = 1/\zeta + 1/\theta$, that

$$\begin{aligned}\mathbf{E}[D_M(T-\tau) \mid \tau < T \le t_{\max}] = d\mathbf{E}[M] &- \frac{d\delta\mathbf{E}[M^2]e^{\tau/\eta}}{\theta^\kappa\chi^\kappa}\frac{\gamma(\kappa, t_{\max}\chi) - \gamma(\kappa, \tau\chi)}{\gamma(\kappa, t_{\max}/\theta) - \gamma(\kappa, \tau/\theta)}\\ &- \frac{d\delta\mathbf{E}[M^2]e^{\tau/\rho}}{\theta^\kappa\omega^\kappa}\frac{\gamma(\kappa, t_{\max}\omega) - \gamma(\kappa, \tau\omega)}{\gamma(\kappa, t_{\max}/\theta) - \gamma(\kappa, \tau/\theta)}\\ &+ \frac{d\delta\mathbf{E}[M^2]e^{\tau/\zeta}}{\theta^\kappa\xi^\kappa}\frac{\gamma(\kappa, t_{\max}\xi) - \gamma(\kappa, \tau\xi)}{\gamma(\kappa, t_{\max}/\theta) - \gamma(\kappa, \tau/\theta)}.\end{aligned} \tag{24}$$

Expected cost $\Delta(2,2)$: conclusion Combining Eqs. (21), (23) and (24), we express (17) as

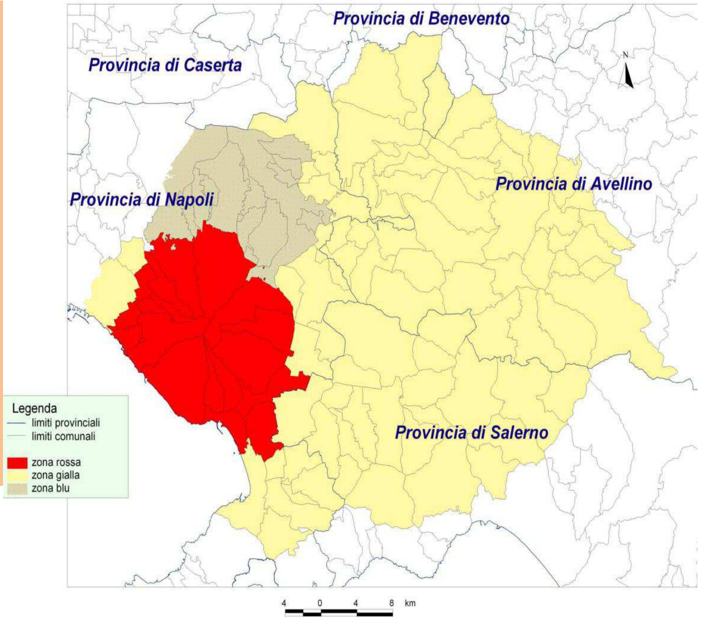
$$\begin{aligned}\Delta(2,2) = \Big(b\rho &- \frac{b\rho e^{\tau/\rho}}{\theta^\kappa\omega^\kappa}\frac{\gamma(\kappa, t_{\max}\omega) - \gamma(\kappa, \tau\omega)}{\gamma(\kappa, t_{\max}/\theta) - \gamma(\kappa, \tau/\theta)} + \frac{\delta c\mathbf{E}[M]e^{\tau/\eta}}{\theta^\kappa\chi^\kappa}\frac{\gamma(\kappa, t_{\max}\chi) - \gamma(\kappa, \tau\chi)}{\gamma(\kappa, t_{\max}/\theta) - \gamma(\kappa, \tau/\theta)}\\ &+ d\mathbf{E}[M] - \frac{d\delta\mathbf{E}[M^2]e^{\tau/\eta}}{\theta^\kappa\chi^\kappa}\frac{\gamma(\kappa, t_{\max}\chi) - \gamma(\kappa, \tau\chi)}{\gamma(\kappa, t_{\max}/\theta) - \gamma(\kappa, \tau/\theta)}\\ &- \frac{d\delta\mathbf{E}[M^2]e^{\tau/\rho}}{\theta^\kappa\omega^\kappa}\frac{\gamma(\kappa, t_{\max}\omega) - \gamma(\kappa, \tau\omega)}{\gamma(\kappa, t_{\max}/\theta) - \gamma(\kappa, \tau/\theta)}\\ &+ \frac{d\delta\mathbf{E}[M^2]e^{\tau/\zeta}}{\theta^\kappa\xi^\kappa}\frac{\gamma(\kappa, t_{\max}\xi) - \gamma(\kappa, \tau\xi)}{\gamma(\kappa, t_{\max}/\theta) - \gamma(\kappa, \tau/\theta)} + a\Big)\frac{\gamma(\kappa, t_{\max}/\theta) - \gamma(\kappa, \tau/\theta)}{\Gamma(\kappa) - \gamma(\kappa, \tau/\theta)}.\end{aligned}$$

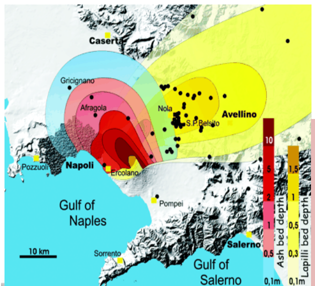
Example: Vesuvius

Evacuation problem extensively studied - excellent for benchmarking

Current emergency plan (Protezione Civile 2014)

- based on subplinian (VEI 4) eruption similar to AD 1631 eruption
  - Pr(VEI 4) ~ 29%
  - Pr(VEI 5+) ~ 1%, 4%, 1-20%, …
- evacuation zones
  - red zone (pyroclastic flow hazard) to be evacuated *prior to eruption* (~ 900,000 people)





Worst Case:
Avellino 3780-yr-B.P.
(Mastrolorenzo et al 2006)

Time to eruption onset

Recall:

T = eruption onset, T ~ Gamma(κ,θ) random variable: $f_T(t) = \dfrac{t^{\kappa-1}\exp(-t/\theta)}{\theta^{\kappa}\Gamma(\kappa)}$

Sandri et al. (2009) applied BET_EF *retrospectively* to AD 1631 eruption, **using 2006 (MESIMEX) settings**

Need to estimate κ,θ as functions of P

Table 2 Application of BET_EF to the 1631 eruption (Sandri et al. 2009); q_z means the 100z% quantile.

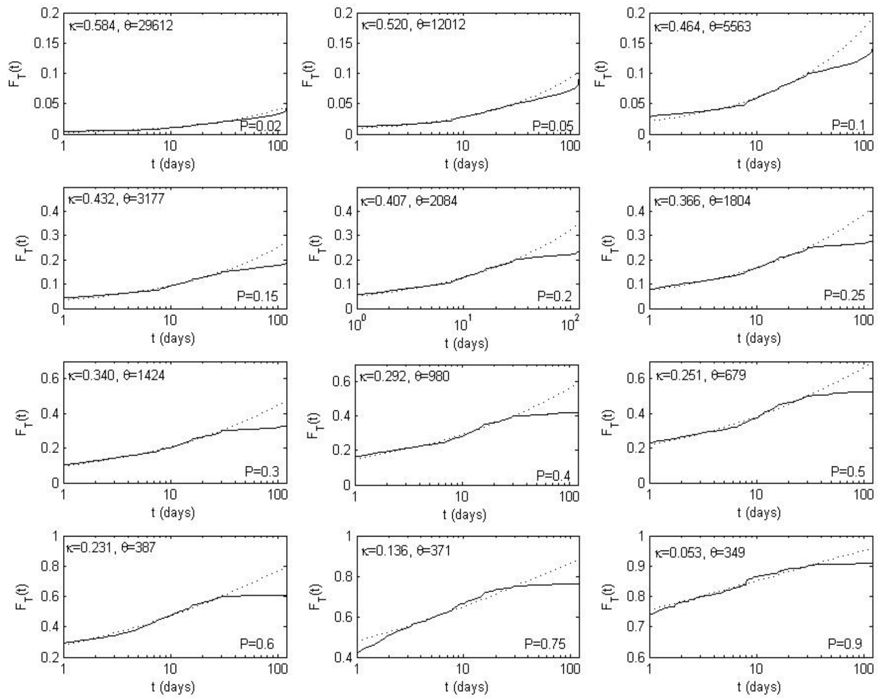
Days to eruption	Absolute Probability of Eruption				Parameters	
	Mean	$q_{0.1}$	$q_{0.5}$	$q_{0.9}$	α	β
120	0.02	0.002	0.02	0.05	1	43
30	0.08	0	0.002	0.29	0.12	1.36
16	0.13	0	0.02	0.50	0.19	1.17
8	0.28	0	0.13	0.83	0.28	0.73
0.33	0.27	0	0.10	0.85	0.23	0.62
0.04	0.77	0.13	0.98	1	0.45	0.13

Distribution of probabilities fitted by Beta distribution

- Gives distribution of probabilities at known time from eruption.
- 'Invert' via Monte Carlo simulation and spline fitting to get distribution of time to eruption given estimated probability of eruption
  - Normalize so that Pr(T<30) = P from BET_EF

Resampled distribution of time to eruption

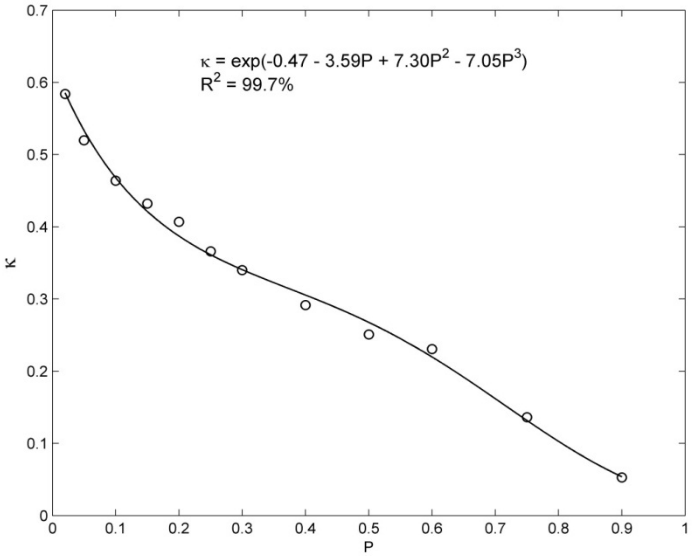
$$f_T(t) = \frac{t^{\kappa-1}\exp(-t/\theta)}{\theta^{\kappa}\Gamma(\kappa)}$$



Resampled (solid line) and fitted gamma (dotted line), distributions for the time to eruption (T) at Vesuvius given an estimated absolute probability of eruption (P).

Fitted distribution for all P via regression

$$f_T(t) = \frac{t^{\kappa-1}\exp(-t/\theta)}{\theta^{\kappa}\Gamma(\kappa)}$$



$\log \kappa = -0.47 - 3.59\,P + 7.3\,P^2 - 7.05\,P^3$

$Pr(T < 30) = P = \gamma(\kappa, 30/\theta) \,/\, \Gamma(\kappa)$ … solve for θ

Get κ, θ and hence $f(t)$ for any P

Costs and other parameters

Costs (per capita):

Disclocation, b = 90 euros/day

Human life, c = 800,000 euros

Displacement, 32,000 euros/year

(Marzocchi & Woo 2009)

Hazard:

δ = 0.1 (10% deaths from reference eruption)

(Neri et al. 2007)

M has exponential distribution, Pr(M>1) = 0.1

Times:

- Average displacement from red zone ~ 1 year due to infrastructure damage (Zuccaro et al. 2013) and new cycle of volcanic activity
- Average time to spontaneous return if no eruption = 130 days (50% return after three months)
- Average time to evacuate = 3.125 days

  (20% unevacuated after 5 days)
  - "within 72 hours and no more than 7 days" (Protezione Civile 2014)

Baseline solutions

Static evacuation threshold
(Marzocchi & Woo 2007; 2009)

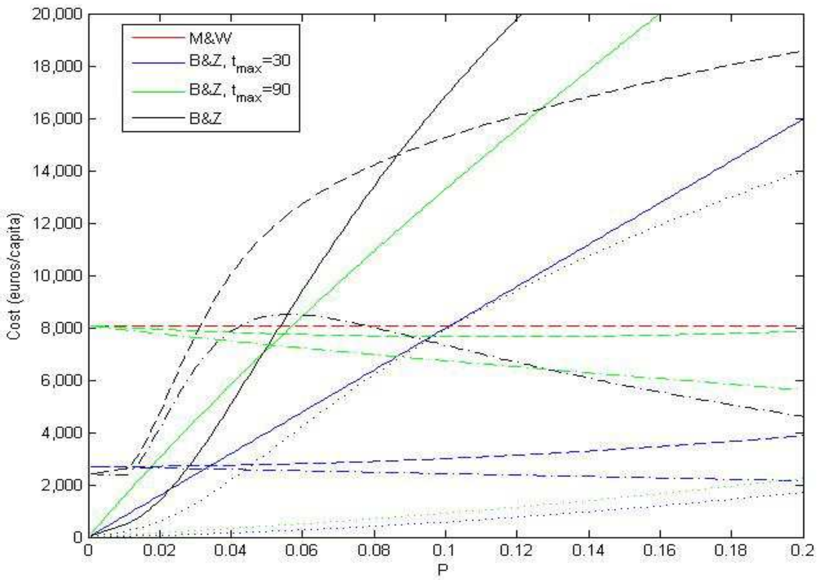
$$P^* = \frac{90b / c}{\delta \bar{M}} = 0.101$$

Evacuation window 90 days
Eruption window 30 days

Dynamic evacuation threshold

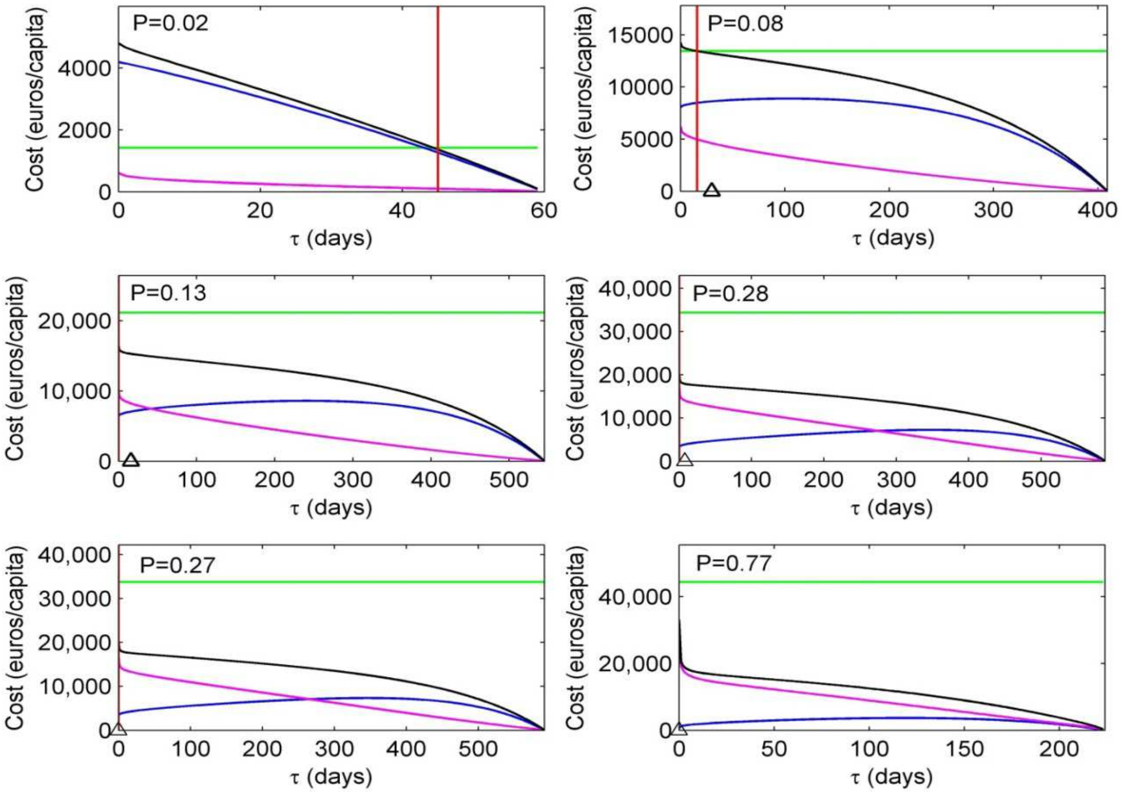
$P^* = 0.087$

Evacuation and eruption windows depend on P



Expected losses from the dynamic model and those of the (Marzocchi & Woo 2007, 2009) static model:
Under no evacuation (solid line), and evacuation (dashed line).
The intersection is the evacuation threshold.
The red and blue solid lines are identical.

AD 1631 redux



Costs are:

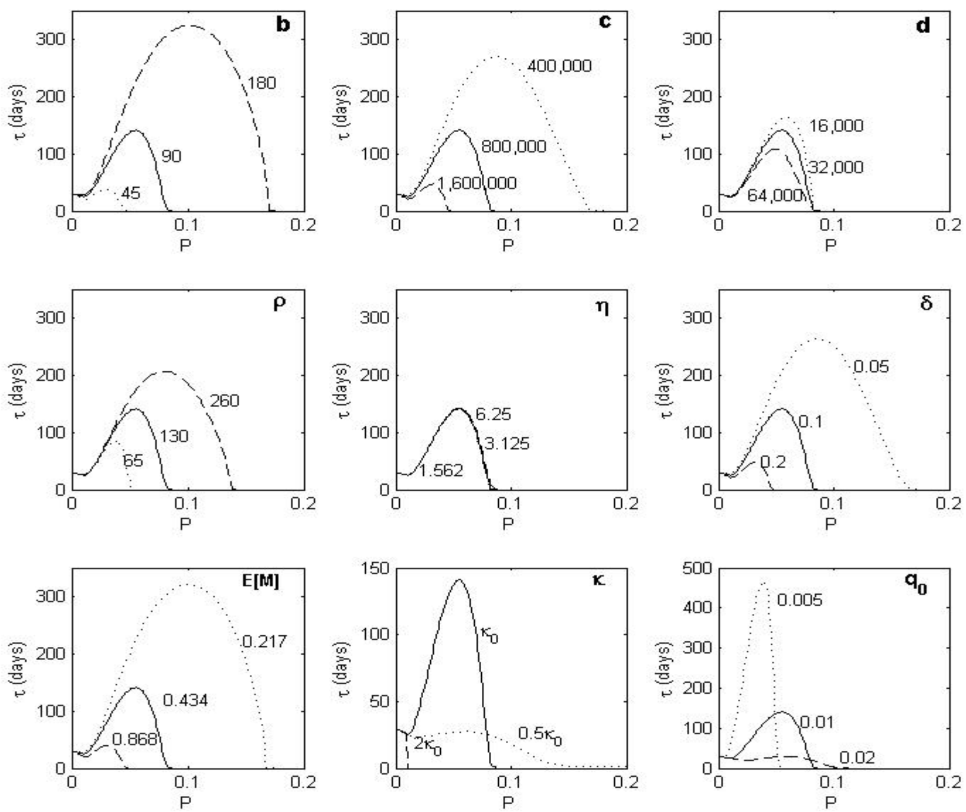
no evacuation (green),
evacuation + no eruption (blue),
evacuation + eruption (magenta)
evacuation (black).

Vertical red line is optimal evacuation time , triangle is the actual eruption.

- No alerts with 0.087 < P < 0.1 so same **decision** as M&W static criterion

- Additional information:
  Advance warning of evacuation (P=0.02, 0.08) and end of alert period (P=0.13)

Sensitivity analysis



Optimal evacuation times τ . The solid line is the baseline. Values (baseline, doubled or halved) of the indicated parameter are shown. The evacuation threshold P ∗ is reached at the x-axis intercept.

<u>Parameter:</u>

κ (uncertainty in time to eruption)

b (dislocation cost)
c (value of human life)
δ (deadliness of hazard)
<M> (relative size of hazard)

ρ (average return time)
q_0 (~window length)

d (displacement cost)
η (average evacuation time)

Increasing sensitivity

Spontaneous evacuation

Spontaneous evacuees are not a cost until (unless?) an evacuation is called. Hence:

Corollary 1 Suppose a proportion (S, say) of the at-risk population has already spontaneously evacuated, then Criterion 1 is satisfied at time t0 + τ if and only if $\tau \le t_{max}$ is such that

$$\Delta(2, 1) + \Delta(2, 2) < (1 - S)\Delta(1, 2)$$

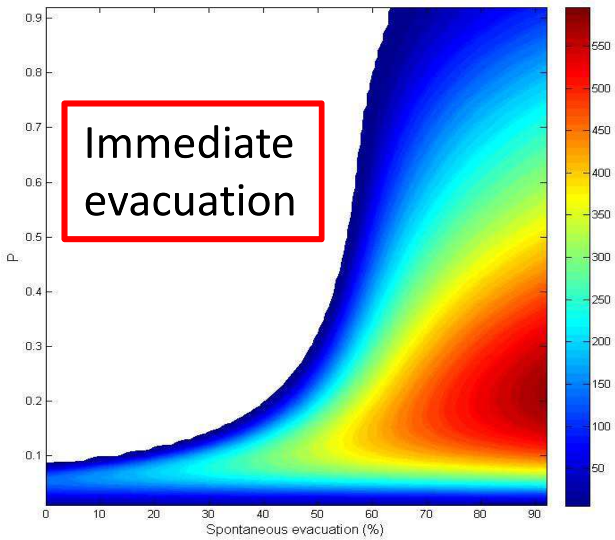
where $\Delta(1, 2)$, $\Delta(2, 1)$ and $\Delta(2, 2)$ are as defined in Theorem 1.

Above S = 0.59, P^* is so large that eruption will occur well before evacuation is complete.

As av. evacuation time ↘0, this limit ↘0.72

Corresponding limit in static case is S = 0.77

This may be economic (CBA), is it ethical?

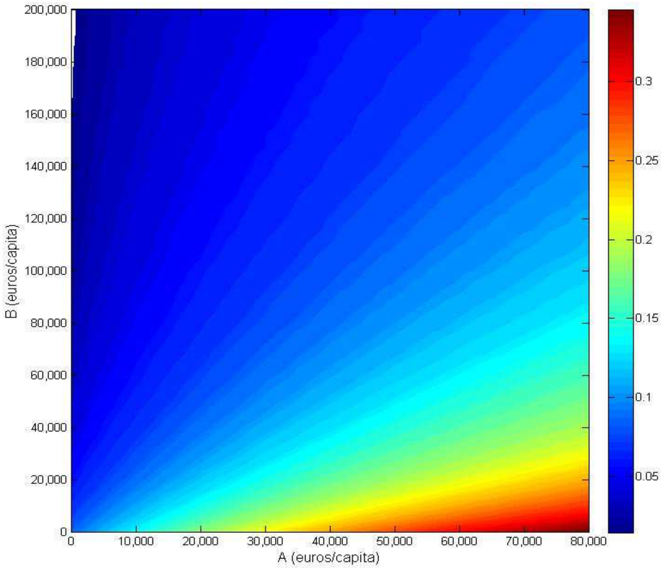


Evacuation warning time τ with spontaneous evacuation. The boundary between the white and blue in the upper left quadrant gives the threshold P^* for immediate evacuation.

Political costs … to the decision maker

The decision maker(s) is expected to get it right in 20/20 hindsight:

A = cost for calling a "needless" evacuation
B = cost for not calling a "needed" evacuation



P^* for an immediate evacuation in the presence of both political costs

"You want me to call an evacuation on a 1 in 10 chance of an evacuation? Come back when it's 1 in 5 …"

- ~ per capita cost of 18,000 euros, or 20,300 lives

"So the expected loss of life is 80,000 euros per capita? Then if I 'subtract that', I can't be arrested … surely?"

- P^* ~ 0.03

P^* invariant provided B ~ 2.25 A

Conclusions

- The volcanology journal doesn’t exist that will publish this paper. ☹
- Chances of it being appreciated by the typical decision maker even smaller ☹ ☹
- Estimates of P from BET_EF etc. are coarse, sporadic and uncertain
  - Simple P* threshold is required message
  - Our results allow the sensitivity of the P* to non-economic factors to be checked, and compared with possible values from BET_EF (or other models)
    - P^* can then be adjusted if necessary

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