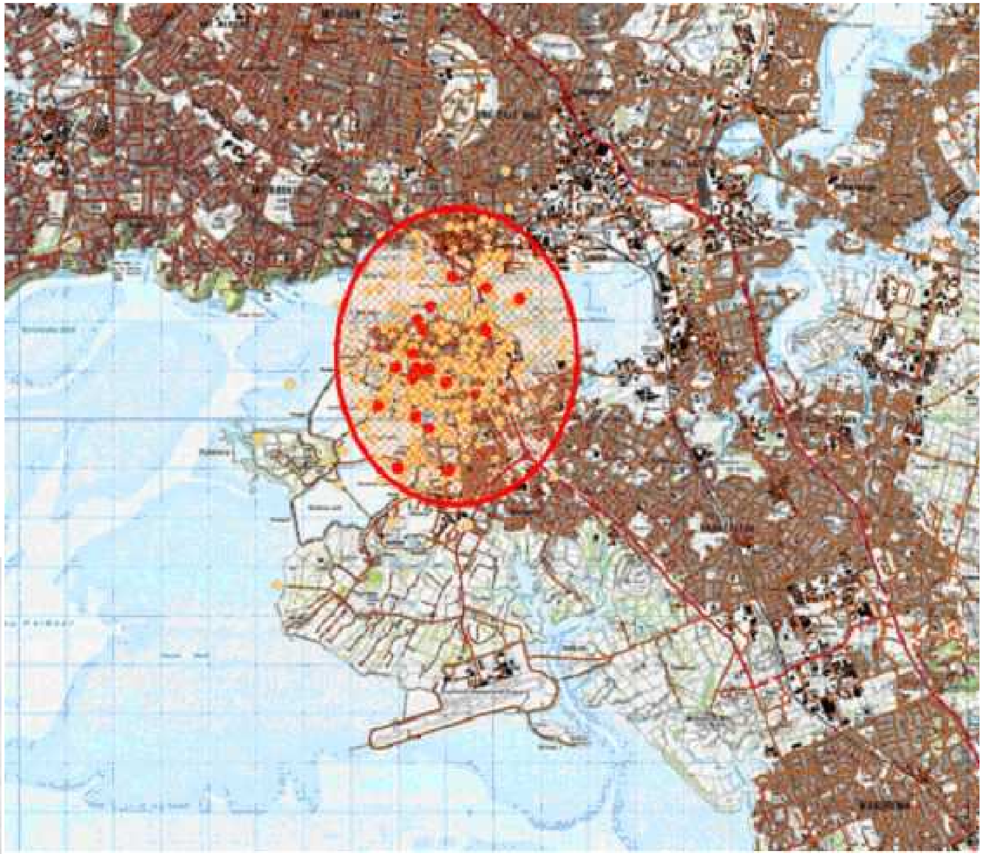
Spatio-volumetric hazard estimation, with an application to the Auckland Volcanic Field



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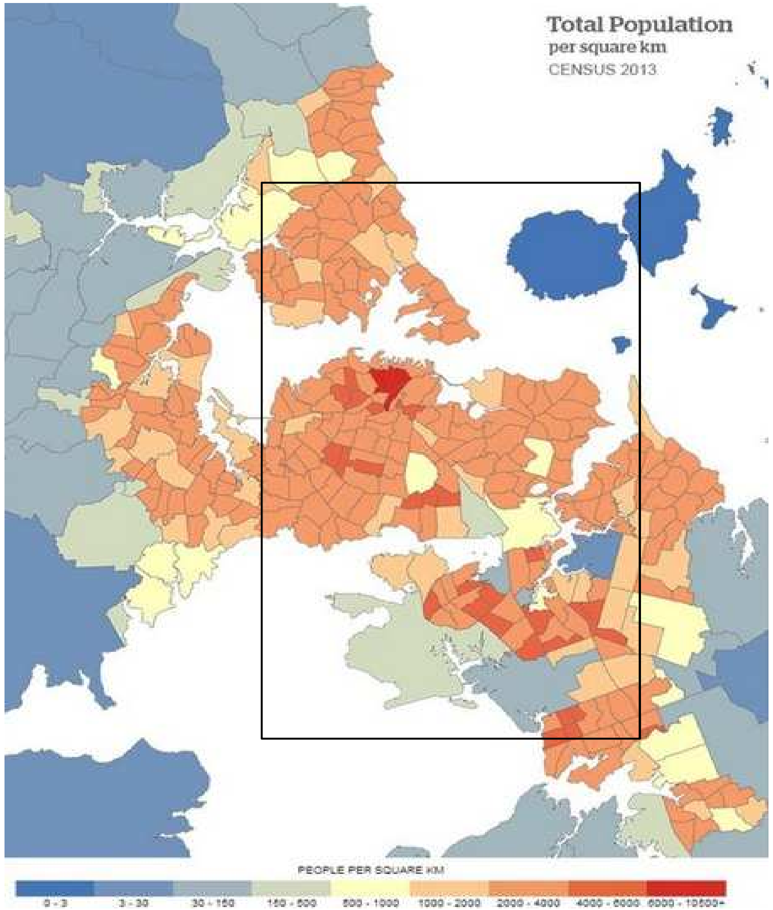
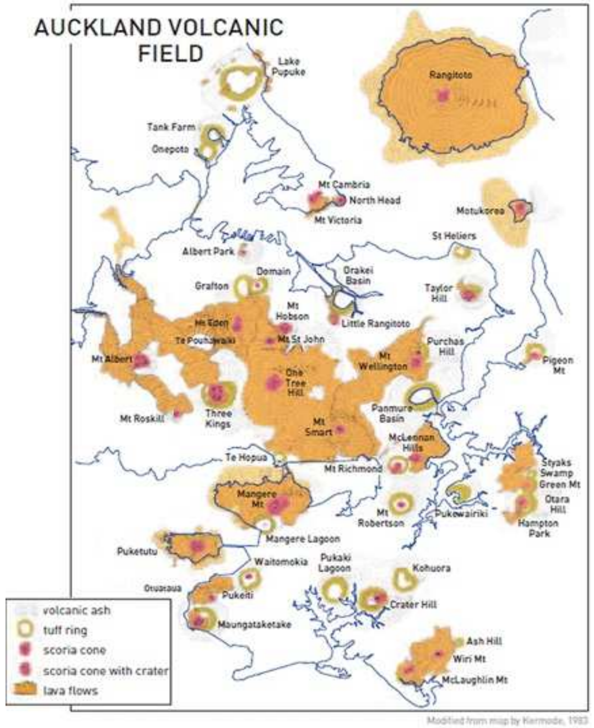
The Auckland Volcanic Field





High population density, Lifelines narrowly constrained

- **<u>WHERE</u>** is the next eruption likely to be?

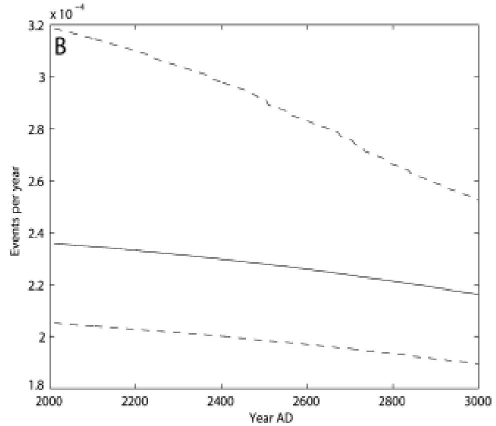
Spatio-temporal hazard estimates

Monogenetic volcanic fields have multiple volcanoes. An eruption likely creates a new volcano.

Probability forecast: estimate the *hazard* $\lambda(t,x)$ such that the probability of an event in the time-space window $(t,t+\Delta t)$ **x** $(y:||y-x|| < \Delta x) \sim \lambda(t,x)\, \Delta t\, \pi\, \Delta x^2$

For a number of reasons, it makes more sense to concentrate on the spatial problem

- Spatial and temporal terms are usually considered independent,
  $$\lambda(t,x) = \nu(t)\eta(x)$$
- Temporal rates are usually low, and don't vary much:
- Spatial data is usually very good, temporal data very poor.



Volume data (Allen and Smith 1994, Kereszturi et al 2013) is also pretty good …

Spatial Intensities : Kernel Densities

Anisotropic Gaussian kernel:

$$\eta(x) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{2\pi\sqrt{|H|}}\exp\left(-0.5(x-x_i)^T H^{-1}(x-x_i)\right)$$

Bandwidths are estimated by leave-one-out cross-validation, (Vere-Jones 1992) maximizing the Kullback-Leibler score:

$$S = \sum_{i=1}^{n}\log\hat{\eta}_{x\backslash x_i}(x_i) - \int_y \hat{\eta}_{x\backslash x_i}(y)\,dy$$

Calculated using all the data except x_i

= 1 if integrated over the support of η

There's something odd about the AVF …

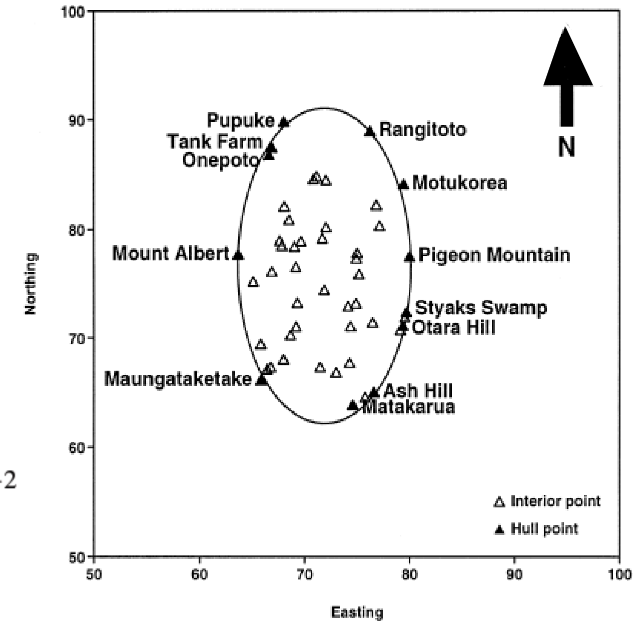
A *lot* of vents lie ***very*** *close* to the minimum area ellipse (Sporli and Eastwood 1997):

Equation of ellipse: $(x-c)^{T}A(x-c)=1$

- Find A,c by method of Khachiyan (1996)

$$A = \begin{bmatrix} 0.0144 & -0.0004 \\ -0.0004 & 0.0045 \end{bmatrix} \text{km}^{-2}$$

with an area of 389.4 km^2.



A valid model for the spatial intensity must *be able to* reproduce this elliptical behaviour

The KISS model for spatial intensity

What about a constant intensity in the ellipse, zero outside:

$$\eta(x) = \begin{cases} 1/\text{Area}, & x : (x-c)^T A\,(x-c) \le 1 \\ 0, & x : (x-c)^T A\,(x-c) > 1 \end{cases}$$

A finite number of observations will underestimate the ellipse extent:

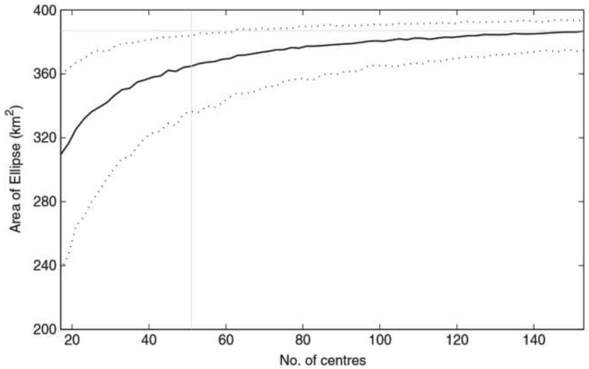


Fig. 2 Areas of minimum area ellipses for varying number of simulated centres. The *solid curve* is the median area, the *dotted curves* show the 90 % interval. The area (389.36 km^2) and number of centres (51) in the AVF are indicated by the *grey lines*

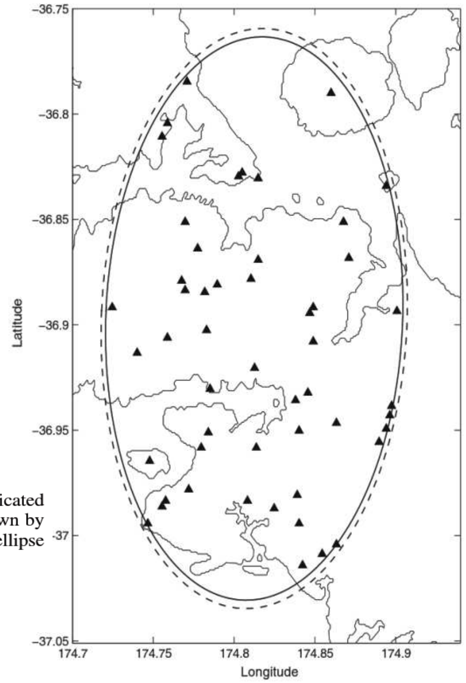


Fig. 1 The Auckland Volcanic Field. Volcanic centres are indicated by *triangles*, with the corresponding minimum area ellipse shown by the *solid curve*. The *dashed curve* is the hypothesized source ellipse denoting the true boundary of the field (see text)

So we need a slightly (~7%) larger ellipse

Elliptical Source v. Anisotropic Kernel

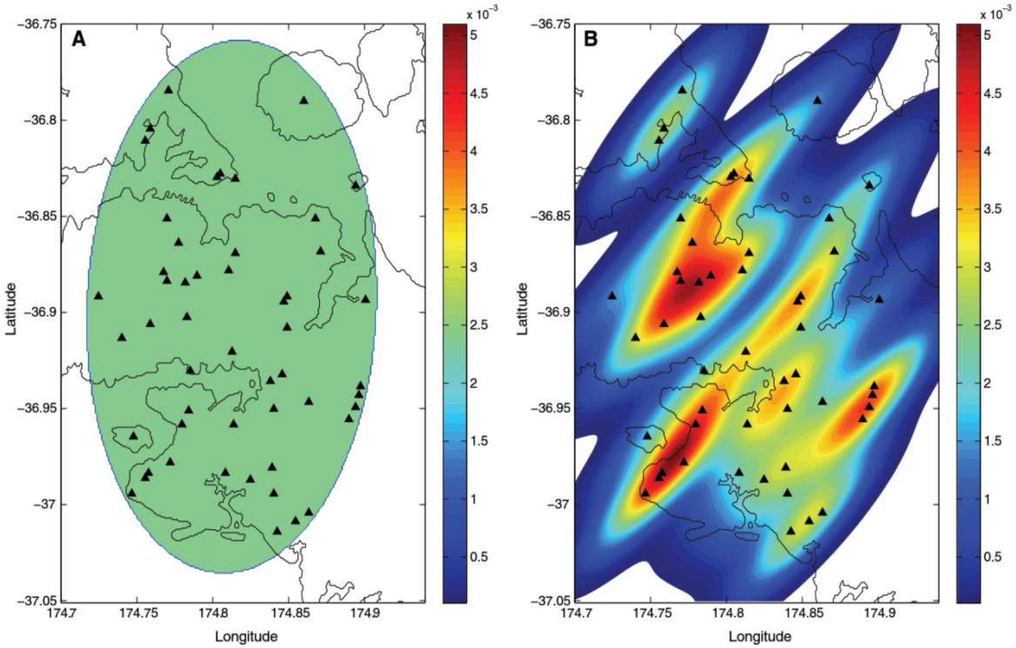


Fig. 3 Spatial densities (km^{-2}). **a** Elliptical source, **b** anisotropic kernel

The anisotropic kernel fits the data much better (higher density at observed points).

BUT … can it produce an elliptical boundary?

A test for clustering …

Look at two characterizations of spatial distribution:

- Clustering, and
- Elliptical boundary

Use Monte-Carlo method:
Simulate repeated samples of size n = 51 from the model, and calculate:

- Interpoint distance distribution (Magill et al. 2005), and
- Distribution of distance to the elliptical boundary (as defined by the simulated set *),

$$D = 1 - (x-c_*)^T A_* (x-c_*), \quad 0 < D < 1$$

- The observed interpoint and boundary distance distributions should be within the 95% intervals of the simulations

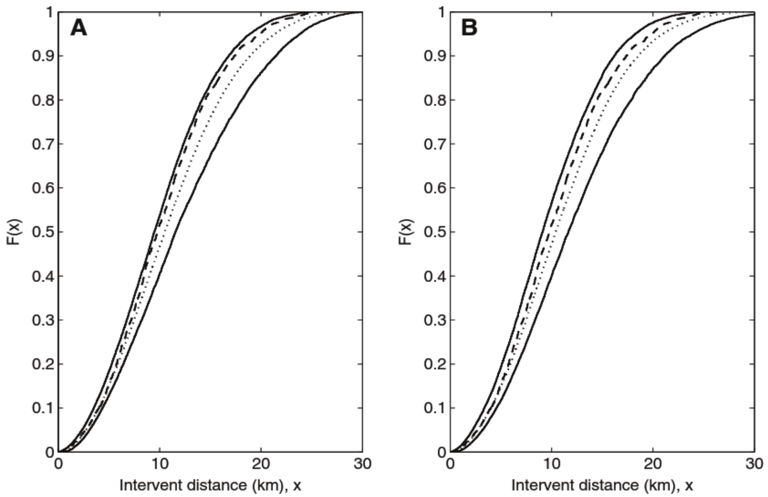


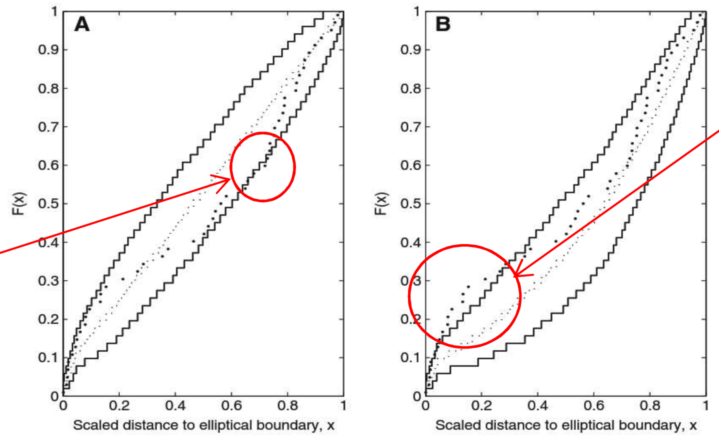
Fig. 4 Inter-point distance distributions. **a** Elliptical source, **b** anisotropic kernel. The *dashed line* is the observed data, while the *dotted and solid lines* show, respectively, the median and 95 % intervals from the model

Both models OK for clustering

… and an elliptical boundary

Both models fail to reproduce the observed boundary behaviour

Fig. 5 Distance (to elliptical boundary) distributions. **a** Elliptical source, **b** anisotropic kernel. The *dots* are the observed data, while the *dotted and solid lines* show, respectively, the median and 95 % intervals from the model



Model doesn't reproduce observed dearth of points at $0.6 < D < 0.7$

More observed points close to boundary than model can produce

Models fail at different places. Can we combine them somehow?

$$\eta(x) = \frac{1}{n 2\pi \sqrt{|H|}} \sum_{i=1}^{n} \frac{1}{M(x_i, H)} \exp\left[-0.5(x - x_i)^T H^{-1}(x - x_i)\right] \quad (10)$$

where

$$M(x_i, H) = \frac{1}{2\pi \sqrt{|H|}} \int_A \exp\left[-0.5(x - x_i)^T H^{-1}(x - x_i)\right] dx \quad (11)$$

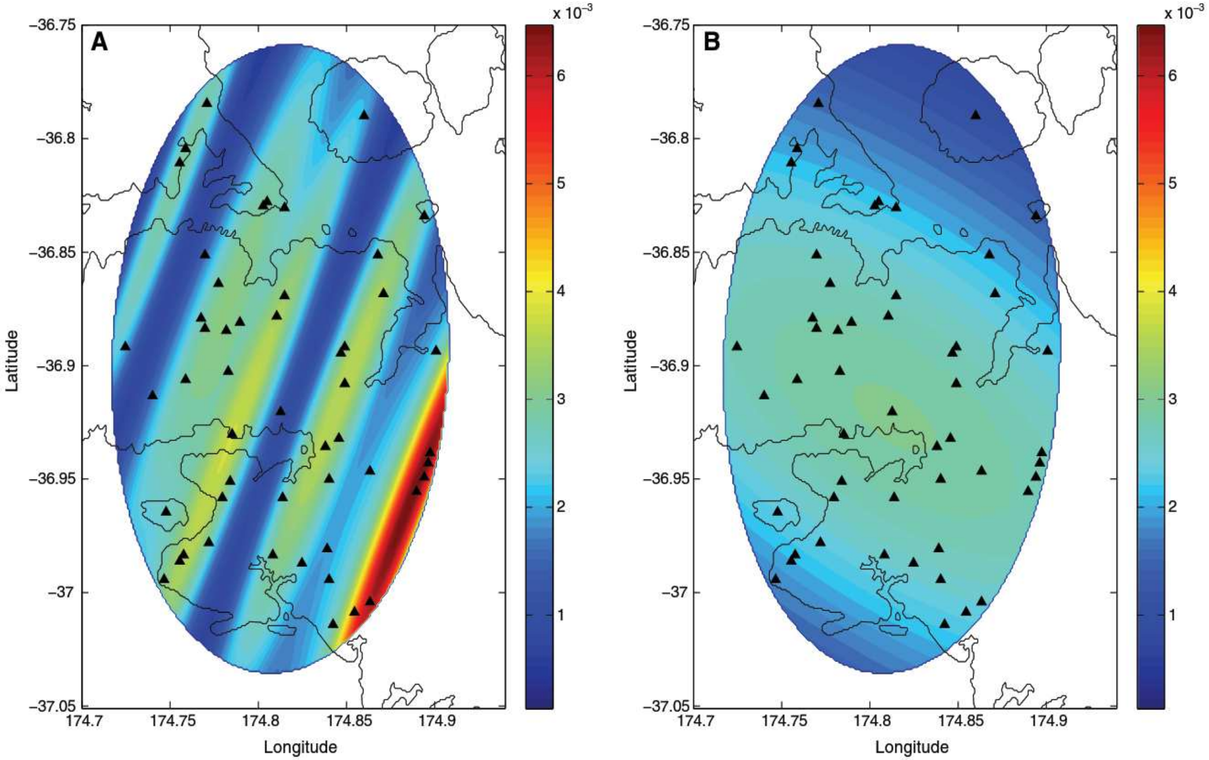
Edge-corrected kernel, reweighted to have total density: 1 inside boundary, 0 outside boundary

Edge corrected anisotropic kernel(s) …

"…it's an ambush – there are two of them!"

S = -305.28

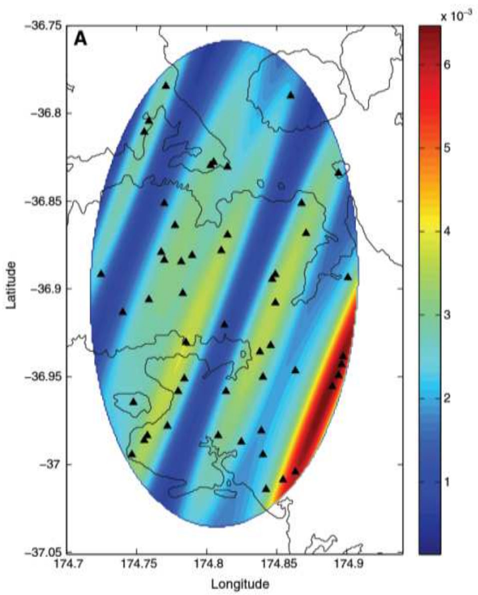
Insignificantly better

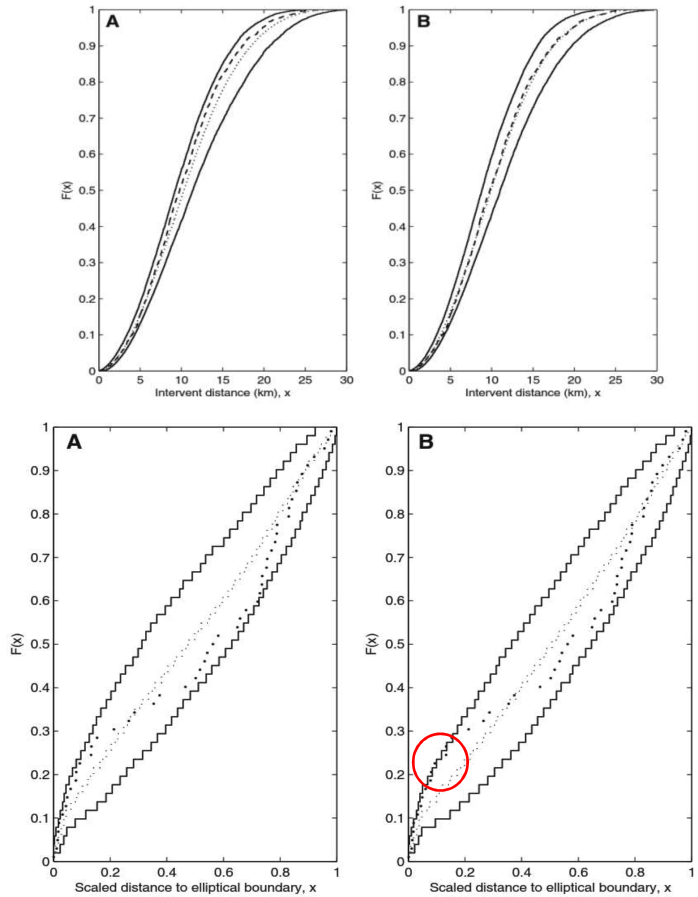


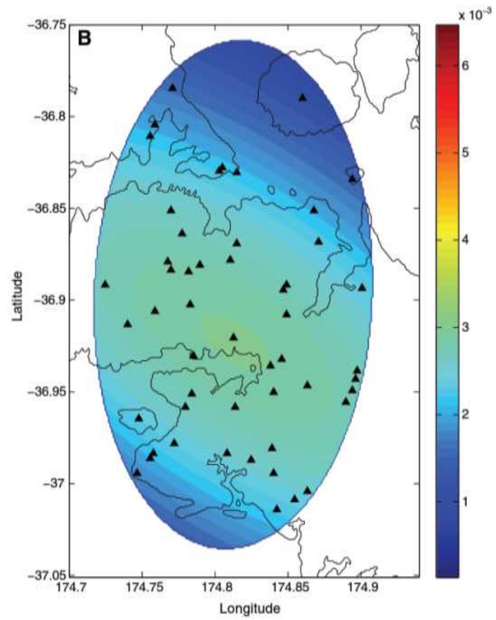
S = -305.30

Fig. 6 Spatial densities (km^{-2}) from two bounded anisotropic kernels. **a** *H* given by Eq. 12, **b** *H* given by Eq. 13

… and their test results







A model for erupted volume

End members:

- Source productivity spatially heterogeneous – large volumes should co-locate
- Productivity spatially homogeneous – eruptions 'tap' the source reducing the likelihood of nearby large eruptions

Suppose at location x, the erupted volume $V(x) = U(x)^3$
where U has a gamma distribution with mean m_x and SD s_x

Fit using leave-one-out cross-validation to factors including:
$d_x^{(i)}$, the distance from x to the ith nearest centre,
$v_x^{(i)}$, the volume of the ith nearest centre to x,
the area of the Voronoi cell including x
the distance from x to the elliptical boundary,
and the averages:

$$\langle d/V \rangle_x = \frac{1}{n} \sum_i d_x^{(i)} / v_x^{(i)},$$

and

$$\langle V/d \rangle_x = \frac{1}{n} \sum_i v_x^{(i)} / d_x^{(i)},$$

$$\log m_x = a_1 \log \langle d/V \rangle_x + a_2 \log d_x^{(1)}, \qquad (15)$$

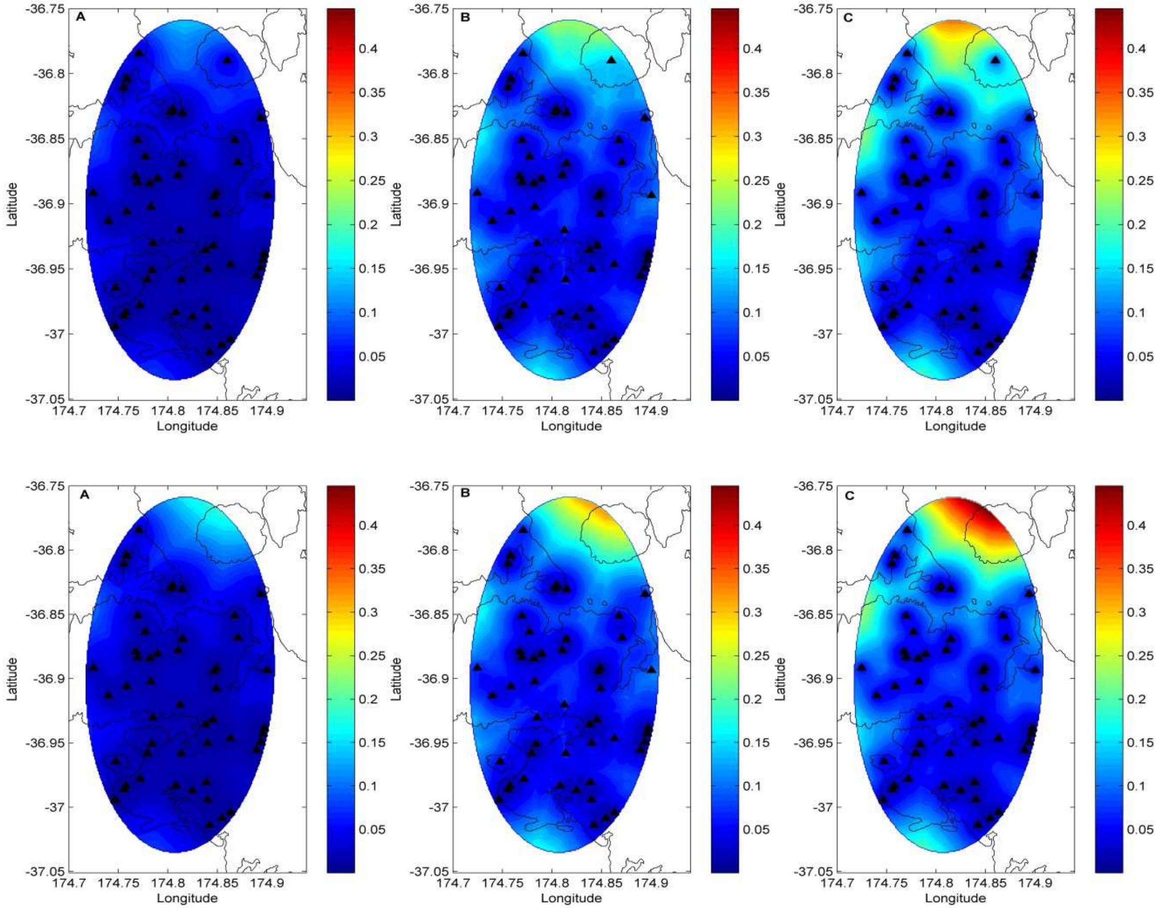
$$\log s_x = a_3 \log d_x^{(2)}, \qquad (16)$$

where $a_1 = 0.483 \pm 0.037$, $a_2 = 0.214 \pm 0.088$ and $a_3 = 0.343 \pm 0.134$ (the uncertainties are given as one standard deviation).

Enough equations! What does it look like?

Present day:

Spatial variation in the forecast volume (in km^3) distribution.
A: Mean,
B: standard deviation,
C: 90th percentile

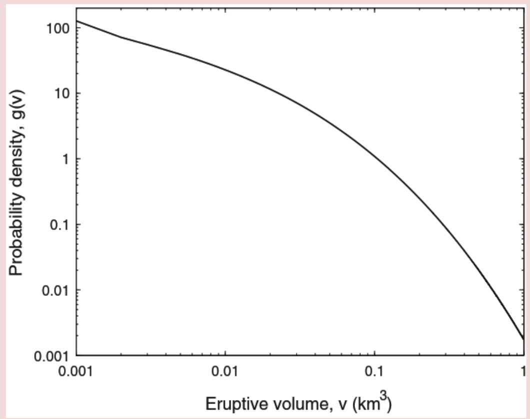


BEFORE Rangitoto:

So that's location and volume … let's put them together

Spatio-volumetric hazard: $\lambda(x,v) = \eta(x)f(v|x)$

Then the probability density for the next volume is: $g(v) = \int_X f(v|x)\eta(x)dx,$



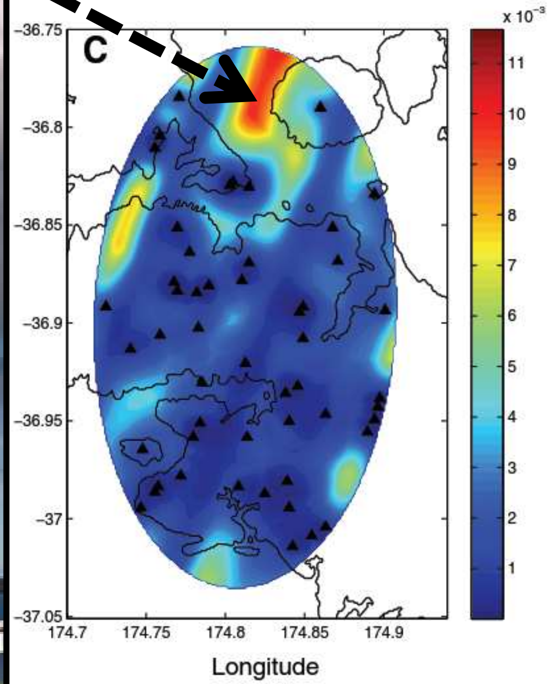
By Bayes Theorem, the location of the next event of volume v is described by the density: $h(x|v) = \frac{f(v|x)\eta(x)}{g(v)}$

More practically, the spatial density for the next vent location, conditional on the erupted volume being between V_0 and V_1, is:

$$\eta_0(x) = \frac{\int_{V_0}^{V_1} h(x|v)g(v)dv}{\int_{V_0}^{V_1} g(v)dv}$$

Maybe the docu-dramatists were right all along …

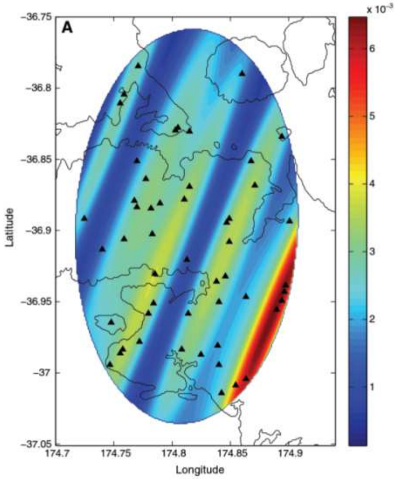
V > $10^8 m^3$



$10^7\ m^3 < V < 10^8\ m^3$, **c** $10^8\ m^3 < V$

Conclusions

- The ellipse-constrained anisotropic kernel is oriented between the NE-SW regional tectonics and the alignments representing the local geology (von Veh and Nemeth 2009)
  - Favours hypothesis (Sporli and Eastwood 1997) of a flat elliptical area in the mantle where tensional stresses allow decompressional melting



- Erupted volumes have no information regarding future locations
  - Suggests no magmatic control on locations, except the boundary
  - The kernel indicates a tectonic control

- Volume and location can be used to forecast volume, but there are no *absolute* location terms in the volume model:
  - Suggests there is magmatic, but not tectonic, control on eruption volumes

$$\begin{aligned} \log m_x &= a_1 \log\langle d/V\rangle_x + a_2 \log d_x^{(1)} \\ \log s_x &= a_3 \log d_x^{(2)}, \end{aligned}$$

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