COMMENT ON "POISSON SCHEMES … "
BY W. ZHU AND M. QIN

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Recently, Zhu and Qin [1] addressed the question of numerically integrating Poisson systems with constant Poisson structure. They concluded that amongst the symplectic Runge-Kutta (RK) methods, only the diagonally implicit ones are Poisson. In fact, they all are, because RK methods are equivariant under linear maps. RK for the Poisson system is therefore equivalent to RK for the system in canonical form with Poisson tensor

$$\begin{pmatrix} 0 & I_m & 0 \\ -I_m & 0 & 0 \\ 0 & 0 & 0_n \end{pmatrix}.$$

For this system, RK leaves the last n variables fixed, so is equivalent to RK for a Hamiltonian system in the first m variables, for which it is a symplectic map. Thus RK for the original system preserves the symplectic leaves and is symplectic on them, as required. This was noted by MacKay in [2].

References

1. W. Zhu and M. Qin, *Poisson schemes for Hamiltonian systems on Poisson manifolds*, Computers Math. Applic. **27** (1994), 7–16.
2. MacKay, R., *Some aspects of the dynamics and numerics of Hamiltonian systems*, The dynamics of numerics and the numerics of dynamics (D. S. Broomhead and A. Iserles, eds.), Oxford, 1992.

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